TOUCHtimes
TASK IDEAS

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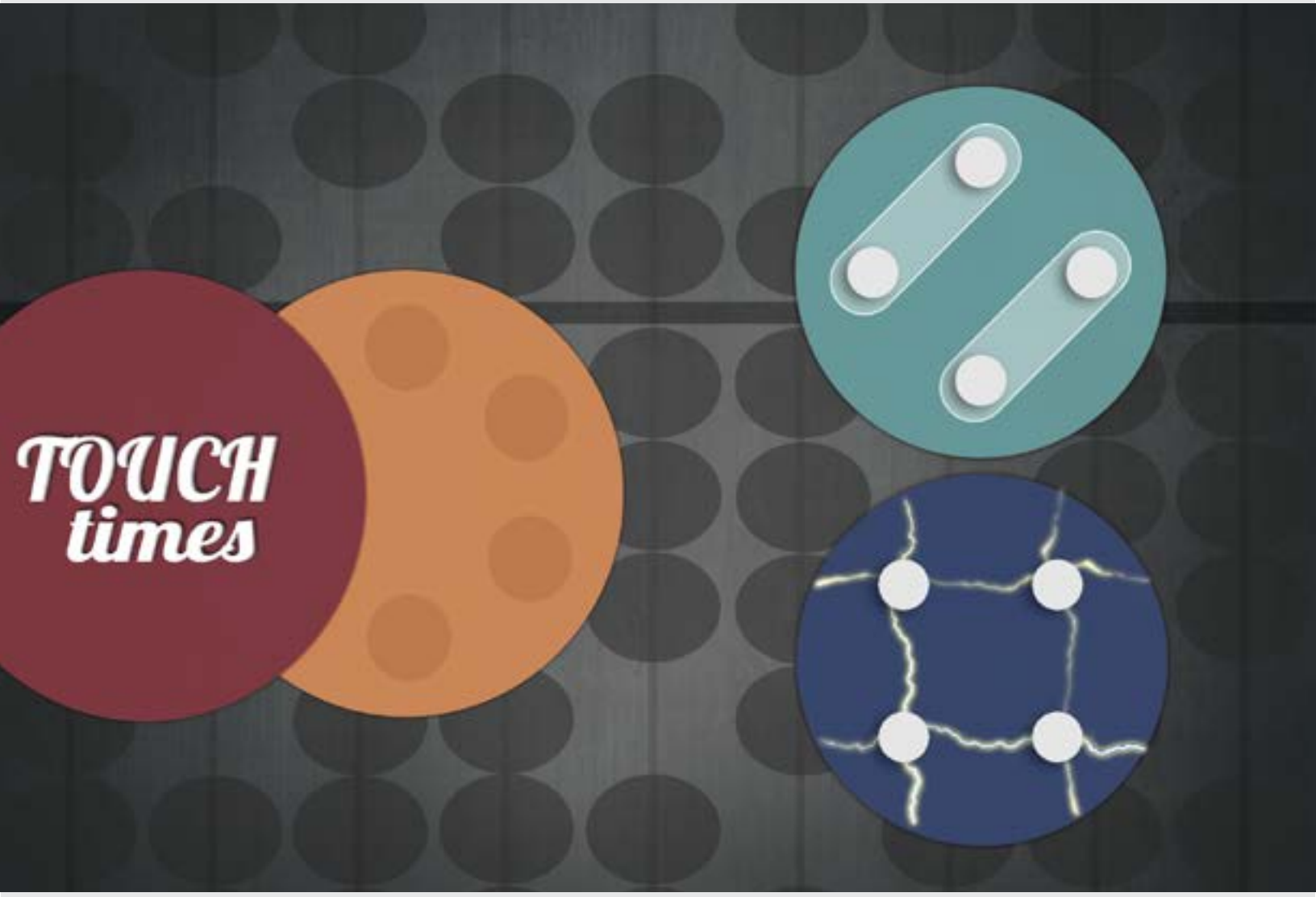

British Columbia Edition

Acknowledgements

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See http://touchcounts.ca/touchtimes/index.html for more information.

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Introduction

Are you looking for new ways to teach multiplication? Are you interested in moving away from procedural approaches that emphasise memorisation and towards conceptual approaches that develop understanding? Are you aware that many students rely heavily on multiplication strategies based in additive thinking and how this might create problems for their future learning?

Multiplication is an important mathematical concept that many children have difficulty learning. Though there are various strategies that students can use when learning their times tables, through the use of *TouchTimes,* we want children to develop an understanding of the underlying multiplicative relationships that are an essential part of multiplication. We do this in part by allowing learners to focus on working with visual and dynamic structures that highlight the relations between factors and their product. *TouchTimes* also uses a novel, tangible approach that engages children through both visual interactions and also kinaesthetic ones. Recent research has shown kinaesthetic interactions with touchscreen technology to be very significant for children's learning. With this in mind, through actions and imagery, *TouchTimes* was developed to help students think multiplicatively, while developing flexibility and fluency with multiplication. Rather than focus on numeric computation and procedural repetition, this learning tool aims to endow children's mathematical ideas with the representational power of their fingers.

What is Multiplication?

Extending beyond counting and additive reasoning, multiplication involves a variety of skills and abilities that centre around relationships of quantity. In order to clarify what multiplicative thinking involves, we outline four key ideas that underly multiplication: **unitising**, **multi-plying**, **covarying** and **spreading**. These ideas overlap in a variety of ways.

Unitising: The idea of **unitising** is an essential multiplicative skill that involves the ability to consider a set of countable items as a single countable item, such as seeing a set of four wheels on a car as a single unit. Multiplication can be thought of as **double unitising**. You first form one unit (the number of wheels on a car) and then **unitise** the number of cars in order to find the total number of wheels.

Multi-plying: Multiplication is an action that simultaneously creates multiple versions of an original unit. Similar to a carbon copy, the original is reproduced or replicated multiple times all-at-once. This contrasts with the action of sequentially adding copies, one after another.

Covarying: As opposed to addition, the quantities in multiplication have a specific predictable relationship with each other. When one factor is varied, the product **covaries** with respect to the other factor. This emphasises multiplication as a varying of two quantities.

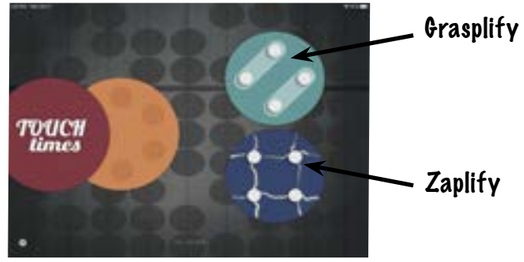
Spreading: This idea emphasises scaling or enlarging. **Spreading** can be seen when a single unit is varied and that variation is seen to **spread** across all other units.

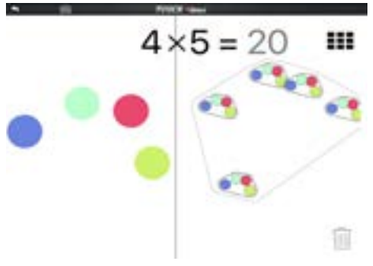
Welcome to TouchTimes

In *TouchTimes*, children use their fingers to multiply. By using two-handed gestures to create their own factors and products, learners develop a strong sense of multiplicative relationships. Since *TouchTimes* displays the product that results from any multiplicative expression, learners focus more on how, using their hands, to co-ordinate and combine multiplicative factors into given multiplicative products. Learners also encounter and work with numbers, symbols and equations, while moving back and forth between symbolic/ numeric forms and more visual, haptic and tactile representations of multiplicative relationships.

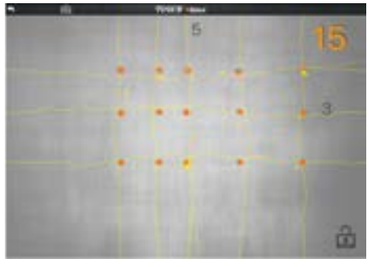
TouchTimes offers two complementary experiences in the Grasplify world and the Zaplify world, each embodying a different multiplicative model. Grasplify is based on a pip-pod (multiplicand-multiplier) model, whereas Zaplify is expressed by orthogonal intersecting lines, similar to an array model. Both worlds involve a two-handed gestural approach for exploring and symbolically expressing **unitising**, **multi-plying**, **covarying** and **spreading**.

A more detailed description of each microworld is in the pages that follow, but experiences with each world allow learners to compare and contrast different ways of thinking about multiplication. Encountering and comparing these similarities and differences is an important part of thinking about multiplication and in order to encourage this process, we explicitly ask the same questions in both worlds. This also provides support to learners who gravitate more towards one world than the other. The experiences that learners have with the two multiplicative models in *TouchTimes* are beneficial in helping them develop more affective relations with different multiplicative models.





Grasplify World



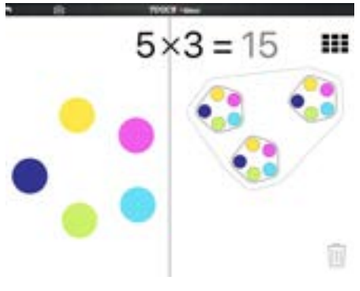
Zaplify World

Grasplify World

The Grasplify world enables experiences of multiplication that go beyond approaches based on additive thinking and repeated addition. In this world, learners experience the **double unitisation** of multiplication. They first create with the fingers of one hand, a unit. The creation of this unit as the first action emphasises **unitising**. Then through one or more simultaneous taps of the other hand—scale that unit to a unit of units, which becomes the product, this is the action of **multi-plying** (as well as the second instance of **unitising**). Grasplify's order follows approaches to early mathematics that are grounded in measurement and ratio, where one wants to identify the unit quantity before asking "how many units?" In research circles, this is known as a Davydovian approach.

Grasplify - Where is the Math?

The Grasplify world emphasises a distinct role for each factor in multiplication, by having learners first create the things to multiply (pips, or more formally: the multiplicand), and then create the number of times by which to **multi-ply** them (pods, or the multiplier). The software displays the total number of pips in all pods as the product of these factors.



Grasplify's ordering of multiplicand-then-multiplier fits naturally with the physical nature of "pips in pods" of the software's design. In more algebraic settings, however, the multiplier frequently appears before the multiplicand, as when we read "5 x *n*" as "five groups of *n*." Each approach makes sense in its context. Thus "5 × 3" can be correctly seen as either "5 groups, each of which contains 3 things" or "5 things, in each of 3 groups." The teacher's choice of language can help support a specific interpretation of such statements in a particular context. Some teachers adopt language such as "5 pips in each of 3 pods"—or more simply "5, 3 times"—to describe Grasplify's physical model.

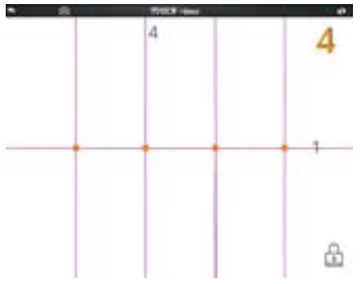
Grasplify embodies **unitising** and **multi-plying**. When one hand makes a set of pips, the learner is **unitising**, and when the other hand makes a set of pods all-at-once, they are **multi-plying**. To draw out the other two key multiplicative ideas, we have designed specific tasks to encourage learners to see that **covarying** the number of pips alters each and every pod, as well as the product. And similarly, **covarying** the pods alters the product. When doing the tasks, we also encourage altering the orientation and number of pips so that learners will see and experience the **spread** of these properties within each pod.

Zaplify World

In Zaplify, learners express multiplication by creating lightning bolts. The number of these vertical and horizontal lightning bolts represent multiplicative factors and the number of orange sparks that appear where they intersect, represent the product. To draw out the key multiplicative ideas in Zaplify, we have designed specific tasks that encourage learners to see the **covarying** of the factors and the effect on the product. In the Zaplify world, **unitising** is created with two hands. If two fingers are touching on the horizontal edge and one finger is touching on the vertical edge, the unit is the intersection of these lines, in this case, four points (4 x 1 or 1 x 4). With additional all-at-once touches on the horizontal or the vertical edges of the screen, intersection points are **multi-plied**.

Zaplify - Where is the Math?

The layout of Zaplify provides learners with a visual grid model—or, more generally, a geometric model—for thinking about multiplication and about how **covarying** factors affect the product. This grid model emphasises factors as being symmetric, such that the order of placing fingers does not matter. In the Zaplify world, children discover that one hand "works" much like the other!



With several fingers along the bottom of the screen producing vertical lines, the horizontal line produced by a finger placed on the left side of the screen will cut across each and every vertical line, thereby **unitising** the set of intersections. Each additional horizontal line **multi-plies** that unit by copying it along each new horizontal line.

The factors and the product are displayed numerically in distinct areas of the screen in order to focus attention on their **covarying** relationships. When a learner makes a change to one factor, the structure of the grid automatically adjusts to reflect the change. This adjustment emphasises the **spreading** of each factor across another factor, that is, when a new vertical line is created, it intersects with each and every horizontal line.

Direct learners' attention to the sparks formed by the intersections of the lightning bolts rather than the spaces created between them. Developing your own name for these—sparks, or fire-bugs, or crashes—helps invest them with identity. As in the Grasplify world, multiplicative products are already computed by the environment, so the focus here should be on patterns and relationships rather than on "what's the product?"

The Benefits of Having Two Worlds

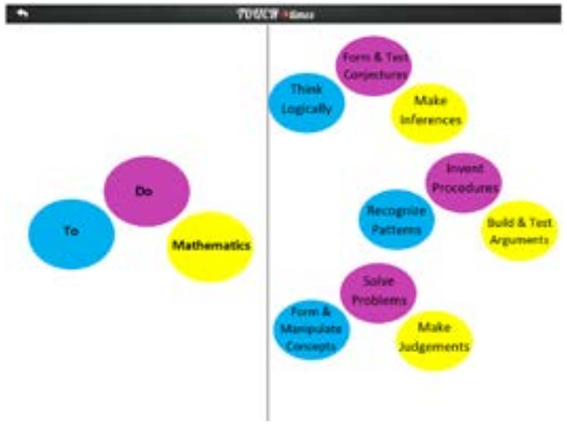
TouchTimes embodies two different multiplicative models. The Grasplify world is based on a pip-pod (multiplicand-multiplier) model and the Zaplify world is expressed by orthogonal intersecting lines, similar to an array model. Both worlds involve a two-handed gestural approach for exploring and symbolically expressing the key ideas of **unitising**, **multiplying**, **covarying** and **spreading** previously described. Both models enable learners to work simultaneously with the physical/concrete and the symbolic/formal aspects of multiplication, instead of moving from the former to the latter.

Some teachers may choose to have their students engage with the Grasplify world first, while others may begin with Zaplify. This choice may depend on the other teaching or learning resources and activities that are being used in conjunction with TouchTimes. Though the Grasplify tasks are first, followed by Zaplify tasks, teachers can choose to work through the tasks in the order given, or can choose to jump back and forth between the tasks--you may even have some students do a task in Grasplify, while others do the equivalent task in Zaplify. The Grasplify tasks are ordered to develop multiplicative thinking step-by-step, while the order of the Zaplify tasks exploits the symmetric nature of multiplication. If you choose to begin with Zaplify to build multiplication step-by-step as in the Grasplify sexion, we suggest the following order of the Zaplify tasks: Zaplify Exporation, Unitising, Doubling, Halving, Skip Counting, Multi-plying, Order Doesn't Matter and Maximising the Product.

Whatever choice you make, we highly recommend a final activity in which you explicitly invite students to compare the two worlds. This might involve asking them to prepare a small video about how each world works, or explaining when or why they think one world is more useful or more interesting than the other.

General Pedagogical Approach

Fundamental to the approach taken in this task booklet is the belief that free exploration and play enable valuable learning opportunities for children as they experiment with mathematical ideas. Mathematics is a form of reasoning that assists in solving meaningful problems. To do mathematics requires more than the performance of rote procedures created by others and memorised, rather it involves examining, manipulating and creating mathematical images and ideas related to quantitative or spatial problems. Using *TouchTimes*, learners create and manipulate mathematical objects with their fingertips, while making sense of what is happening on the screen. Engaging in these processes supports mathematical thinking.

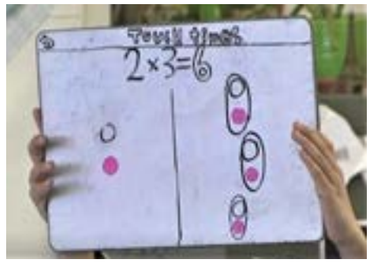


Why Work in Partners?

When children work in partners, they need to co-ordinate their actions on the screen, verbalise their strategies to each other and reflect on what they and their partner are doing. These sorts of skills are outlined in the core communication competency as forms of communication and collaboration. It is documented within that core competency that these kinds of interactions between students support acquiring, developing and transforming ideas and information, which thereby supports learning. Another advantage of working in pairs, is that it increases the number of fingers available to be used!

Getting Started

Children benefit from making choices and having a voice. *TouchTimes* encourages exploration and choice-making in an environment that is structured by mathematical rules but is more open-ended in design. The tasks in this booklet are designed to inspire curiosity and provide learners with opportunities to explore and seek solutions. The role of the teacher involves establishing a spirit of inquiry and supporting learners' experiences with *TouchTimes* by providing time for open exploration, while asking children to predict or explain *TouchTimes'* responses in different situations. Encouraging learners to explain their ideas, as well as demonstrate them, is important, as talking through one's own thinking is one of the strongest ways to develop and solidify understanding. You might try to pose small challenges for them, or to ask them questions about the mathematical challenges and goals that they set for themselves.



Once learners have had sufficient opportunity to solve and make sense of the assigned task, the teacher then brings the whole class together. This provides an opportunity for children to demonstrate, explain and justify their mathematical thinking for their peers, and allows the teacher to introduce or review mathematical concepts and terminology so that learners can use these ideas or vocabularly to communicate and to reflect on their developing ideas in subsequent lessons.

Please note that the tasks can be explored in any sequence you choose, however, be aware that many of the introductory activities are designed to review ideas developed in the previous task.

Connections to Curriculum

The activities in this booklet are designed to connect and relate with each of the Big Ideas and Curricular Competencies listed below. Teachers can narrow in on some or all of the Curricular Competencies depending on the focus for that particular class. [Note that content, content indicators and connections to Indigenizing the curriculum are embedded in each activity write-up.]

Big Ideas

Computational Fluency: (unitising, multi-plying)

- Development of computational fluency in addition, subtraction, multiplication, and division of whole numbers requires flexible decomposing and composing. [Grade 3]
- Development of computational fluency and multiplicative thinking require analysis of patterns and relations in multiplication and division. [Grade 4]

Patterning: (covarying, spreading)

- Regular increases and decreases and patterns can be identified and used to make generalisations. [Grade 3]
- Regular changes in patterns can be identified and represented using tools and tables. [Grade 4]

Curricular Competencies

Reasoning and Analysing

- Use reasoning to explore and make connections
- Develop mental math strategies and abilities to make sense of quantities
- Use technology to explore mathematics
- Model mathematics in contextualised experiences

Understanding and Solving

- Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving
- Visualise to explore mathematical concepts

Communicating and Representing

- Communicate mathematical thinking in many ways
- Represent mathematical ideas in concrete, pictorial and symbolic forms

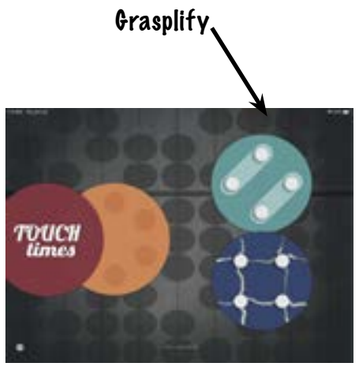
Connecting and Reflecting

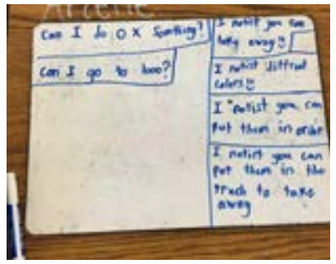
- Incorporate First Peoples worldviews and perspectives to make connections to mathematical concepts (see elaboration in each activity).

GRASPLIFY EXPLORATION

Exploration

Provide time for students to explore the Grasplify world. Have them pay attention to what is happening and jot down things they notice or questions they have. Mini whiteboards are a great resource for students to do this.



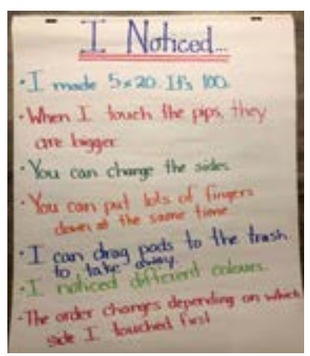


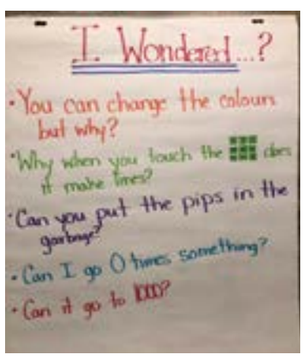
Vocabulary Alert! Watch for opportunities to introduce and reinforce the terms pips and pods.

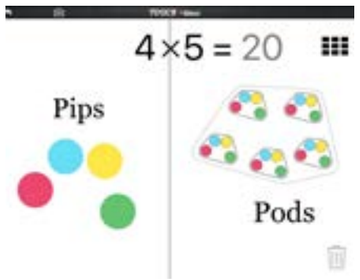
Questions to Ask:

- *What happened when you placed your fingers down on right/left side first?*
- *What do you notice about the number sentence at the top of the screen? How did you make that happen?*
- *Can you place multiple fingers down at once (on the pip side/on the pod side)? What things did you notice about the colours?*
- *What did you notice about Grasplify?*
- *Is there anything that you wonder about Grasplify?*

As students share their ideas, record these on an I Noticed/I Wonder chart for all to see.







VOCABULARY

The pips represent the group size (multiplicand).

The pods represent the number of groups (multiplier).

ZAPLIFY EXPLORATION

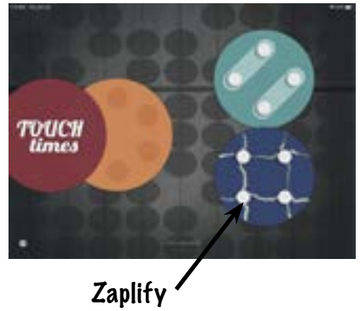
Exploration

Provide time for students to explore the Zaplify world. Have them pay attention to what is happening and jot down things they notice or questions they have. Mini whiteboards are a great resource for students to do this.

Vocabulary Alert! Watch for opportunities to introduce and reinforce the terms in the vocabulary box.

Questions to Ask:

- What happened when you placed your fingers down on the side first?
- What happened when you placed your fingers on the bottom first?
- What do you notice about the numerals on the side and at the top of the screen? How did you make that happen?
- Can you place multiple fingers down at once (on the side/on the bottom)? What happens when you do that?
- What things did you notice about the intersections?
- What did you notice about Zaplify?
- Is there anything that you wonder about Zaplify?
- Encourage students to try different combinations of actions with their hands.



VOCABULARY

The number of lightning bolts from the side represent a factor.

The number of lightning bolts from the bottom represent the other factor.

The total number of intersection points represent the product.

TEACHER TIP

The term factor erases the separation between the multiplicand and the multiplier and assigns identical properties to each unit of count.

TOUCH times

Pips into Pods

Video Link

SETTINGS

For this task, make sure that the Numeric Factor Values and the Numeric Product Values are turned off in settings.

TEACHER TIP

The language used here is very important and highlights the unitising of the pips into a pod and then the unitising of the pods into the envelope.

Summary

This task invites students to notice the unitising relationship between the pips and each pod and the pods and the product.

Game

Choose one student to leave the room and another student to come up to the iPad that is being projected for all to see. Using Grasplify, student 2 creates a multiplication number sentence with pips and pods for all to see. Student 1 returns to the classroom and must recreate student 2's multiplication number sentence based on hints given by classmates.

What to Watch For

- Students who have had experience with multiplication may describe what they see as "4 pods of 5 pips". Encourage them to describe what they see using the pips first, "5 pips taken 4 times" or "5, 4 times" or "5 pips in each of 4 pods". If students comment on the total number of pips in the pods, you can bring the product—20 in this case—into the description.
- Students might also comment on the colour and shape of the pods with respect to the pips, but if they don't, you can prompt their noticing either by asking about colour or by lifting a pip-finger and pressing it down again.

Questions to Ask

- *How did you figure out the total?*
- *Are there are other ways of counting?*

Tasks

1. Ask students to model the following situation using Grasplify: ***On the weekend, I went to the Dollar store to buy some pens. When I got there, they only had packages of 5. I needed enough for our class, so I bought 6 packs. Use Grasplify to show what the packages of pens would look like. Be sure to include enough packages for our class.***
2. Ask students to draw the following situation, ***While at the Dollar store, I also saw some boxes of dry erase markers. There were 4 markers in each box, so I bought 3 boxes. Draw 4 x 3 = 12 using pips and pods the way that Grasplify would show it.***
3. Show a screenshot of Graspify that has 9 pips and 3 pods. Ask students to come up with other situation stories that fit this model. You can vary the numbers used to provide other examples, if needed.
4. Have students create a multiplication story that their partner demonstrates using Grasplify. Challenge students to come up with a variety of different stories.

Questions to Ask

- *How can we find the total number of pens (in this case, pips), without counting every pip?*
- *How does an extra pip (or pen) change the situation in the packages?*
- *What would it look like in Grasplify if each pack contained a different number of pens?*

Extending Student Learning

Provide a list of skip counting number sequences (e.g. 2, 4, 6, 8; 3, 6, 9, 12; 4, 8, 12, 16; 5, 10, 15, 20, 25) and ask students which number sequence they would use to count the pips without counting each and every pip. This way the students who are not fluent in skip counting number sequences might also find an entry point for this task.

Assessment

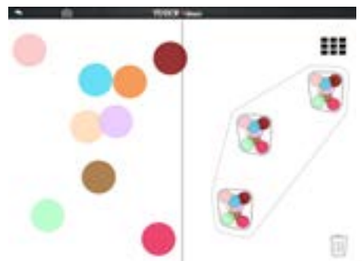
Have students draw a picture of what the Grasplify screen would look like for the pen-package situation and then write a number sentence that reflects what is happening on the screen.

TEACHER TIP

Make sure that students have made 5 pips and 6 pods. Some will have done the inverse. We want students to notice that by skip counting by five, we can find the total number of pens more quickly and easily than by counting each individual pen.

CAUTION!

This visual relationship requires work. Often what children see is not the same thing that we see. Watch to see that they have drawn 4 pips in 3 pods as this is the multiplicative structure in Grasplify.



TOUCH times

Doubling

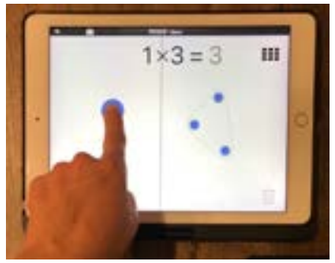
Video Link

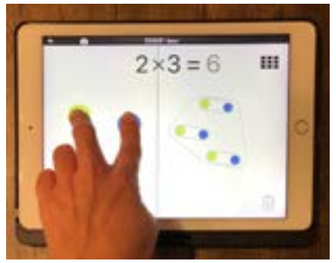
Summary

Students create pictorial representations of multiplication using Grasplify to investigate the mathematical strategy of doubling, which is more multiplicative in nature than repeated addition.

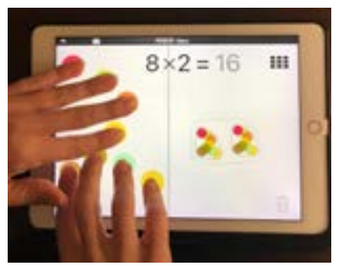
Tasks

1. Project Grasplify for students to see. Make the product 1 × 3 = 3 using one pip and three pods. Challenge students to double the product to six without changing the number of pods, they can ONLY change the pips. Once done, they can double the product again to make 12, then 24.One way of doubling is to place an additional pip-finger on the screen. In the example below, the product doubles, going from 3 to 6.





2. Create 1 x 2 = 2 for students to see. Challenge them to double the product so that it is four by only changing the pips. Once they've done that, have them double it again to make eight, then double that to make sixteen. What will the final equation be?



What to Watch For

- Ensure that pairs have doubled the product by changing the pips, which is multiplicative rather than repeated addition.
- Ensure that children explain what happened within the pods when another pip was created.

Questions to Ask:

- *What happened when you increased the number of pips?*
- *What colour is the pip that you just made? Where else do you see pips that colour?*
- *Do you notice anything about the shape of the pips?*
- *What happens when you push the array button?*

- *Were we adding another group when we increased the number of pips to double the product? What was happening instead that caused the product to double?*

Extending Student Learning

- Give more doubling variations to early finishers.
- Invite students to triple the product. They will quickly run out of fingers, so it will be necessary to imagine how to triple the product and they can document their thinking on paper.
- In a move towards the more general, ask students if they had an unknown number of pips and 4 pods, what would they do to double?

Assessment

Show or explain how to solve this problem using Grasplify:

There are six people sharing a snack. Each person has two strawberries. How many strawberries would there be in total if you doubled the number of strawberries each person got?

TEACHER TIP

This is a great opportunity to introduce students to the array button and explain what arrays are.

VOCABULARY

An array is an arrangement of objects in rows and columns.

TEACHER TIP

This increase in the number of pips should draw attention to the spread effect of pips within each and every pod.

TEACHER TIP

This can be shown either by taking a screenshot or through drawing. Some students will do 6 x 2 = 12, since that number comes first in the problem, though it makes more sense to do 2 x 6 = 12 and then double the two to create 4 x 6 = 24.

Halving

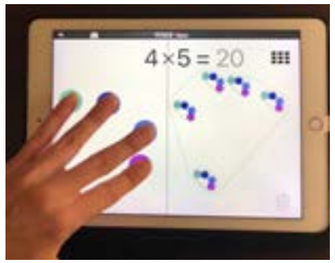
Video Link

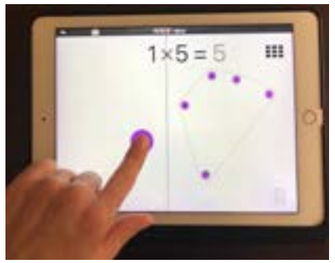
Summary

Students create pictorial representations of multiplication using Grasplify to investigate the mathematical strategy of halving, which is more multiplicative in nature than repeated addition.

Tasks

1. Project 4 × 5 = 20 for students to see. Explain that the product is twenty. Ask students to halve the product by only changing the pips. Once done, halve the product again to make five.





2. As pairs successfully complete the given task. Challenge them to find other ways to halve the product.

What to Watch For

- Ensure that pairs have halved the product by changing the pips, which is multiplicative rather than repeated addition. When first given this task, many pairs halve the product by dragging pods to the trash.
- Students will have to shift from adding more pip fingers to removing half their pip-fingers each time.
- Pay attention to the "choreography" students are using. For example, if a student places 3 fingers from her left hand as well as 3 fingers from her right hand on the pip-side, she can halve the product by lifting one hand. Similarly, if two students each have four fingers on the pip-side, they can halve the product if one of the students takes her fingers off.

TEACHER TIP

Instead of starting from the left, you can make 20 = 5 x 4 by starting on the right side of the screen. It's a good idea to use both directions to help students see that the equal sign does not always have to be on the right. This also provides an opportunity to discuss the meaning of the equal sign with students.

Questions to Ask

- *What do you have to do next in order to halve the product so that it becomes ten?*
- *Which side do you need to lift your fingers?*
- *How is doubling different than halving?*
- *Did you notice anything about halving that is similar to doubling?*

Extending Student Learning

- Early finishers can be challenged to try halving the product of 8 × 5 = 40 by only changing the pips. Prompt them to try halving 40 = 5 x 8.
- Students can also be challenged to halve 10 × 5 = 50 which would quickly produce a problem (halving 25).
- Be sure to try an challenging question like 3 × 7, and ask about halving.
- In a move towards the more general, ask students if they had an unknown number of pips and 4 pods, what would they do to halve?

Assessment

1. Rekha has made 32 = 4 x 8 in Grasplify. Show or explain two different ways she could halve that product to get 16.
2. Draw a picture or explain in words why halving is the opposite of doubling.

TOUCH times

Many-to-One – Part A

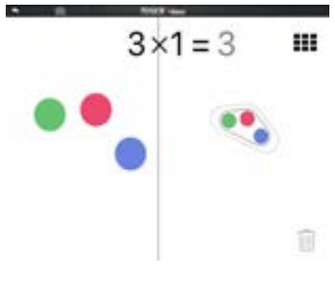
Video Link

Summary

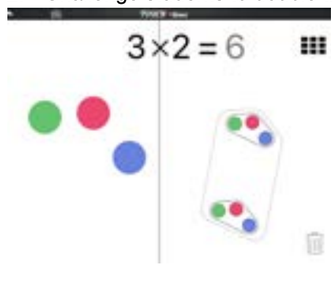
The process of multiplication involves two stages of unitisation (combining many parts into one whole) to get a product. Students investigate how the placement of their fingers when using Grasplify can create a many-to-one unit.

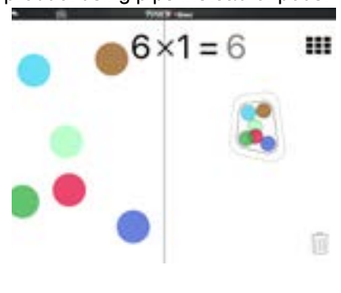
Tasks

1. Display the image on the left. Explain that this is one pod of three and ask students to make two pods of three.



2. Challenge students to double the product using pips instead of pods.





3. Students must now figure out how to make a single pod of five, which can also be called a 5-pod.

TEACHER TIP

The goal of this task is for students to develop fluency in performing the unitising action with different numbers, using only 1 pod-finger.

CAUTION!

The many-to-one concept appears easy but children do not initially find this easy to do or easy to understand.

What to Watch For

- Students must generate 5 pips first in order to create a single pod of 5 with one touch.
- It is common for students to initially place 1 pip-finger down and then tap 5 times sequentially with a pod-finger making five 1-pods instead of one 5-pod (see Figure a).
- If this strategy persists, invite children to try placing several pip-fingers on the screen simultaneously.
- Depending on which side of the screen students tap first, their number sentence may reflect 5 × 1 = 5 or 5 = 1 × 5 (Figure b). Either of which is a successful creation of a 5-pod.

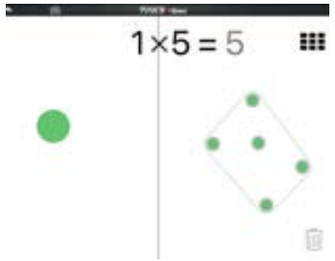


Figure a: One-to-Many
Five 1-pod finger touches

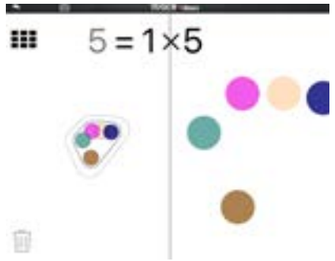


Figure b: Many-to-one
One 5-pod finger touch

Questions to Ask

- *How did you double? How is doubling with the pips different than doubling the pods?*
- *You have five, what do you notice about the colours in the pod?*
- *Did you get this right away? If not, what did you do first? How did you know that was incorrect? How did you figure out how to create a 5-pod instead of five 1-pods?*
- *What will happen if we put one more pip down? What happened to the shape of the pod? What do you notice about the colours?*
- *If you put one pip down and then take one off, what will happen to the colour and the shape?*
- *5 × 1 = 5 and 5 = 1 × 5 are both ways of creating a single 5-pod. Does the order of the number sentence in this case matter? Why or why not?*

Extending Student Learning

- How can you make three with one finger? How can you create a single pod of seven?

Assessment

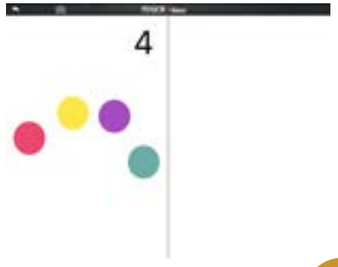
1. Using the image on the right, ask students to draw what the right side of the screen would look like if one finger was put down there.
2. Show how to make 1 bag of 6 marbles in Grasplify.

TEACHER TIP

Make explicit that the 5 represents the 5 pips and the 1 represents 1 pod, which in turn produces 5 (product).

TEACHER TIP

An important part of multiplication involves seeing how the colours work (spreading) and the various aspects of the co-ordination of quantities (unitising and multi-plying) that relate to both colour and shape.



Many-to-One – Part B

Video Link

Summary

The process of multiplication involves two stages of **unitisation** (combining many parts into one whole) to get a product. Students investigate how the placement of their fingers when using Grasplify can create a many-to-one unit. Students are meant to discover that *n* pips must first be established in order for a single *n*-pod to be created. For example, in the photo to the left, four pips are established and then five 4-pods are created with each touch.

Tasks

1. Challenge students to make the largest pod possible.
2. Students will complete the next activity with pencil crayons and paper. Explain that, ***Lucas puts six pip-fingers down on the screen, like this. If his partner Benedict puts one pod-finger down to create a pod, draw what their pod will look like?***

What to Watch For

- Do students pay attention to the shape and colours of the pods?
- We want students to notice not only the **unitisation** but also the relationship between the pips on one side and the colour and shape of the pod configuration on the other side, as noticing this relation is crucial for seeing the **multi-plying**.
- This is a good time to consolidate the vocabulary of pips and pods.

TEACHER TIP

- The point of this task is for students to discover the limits of the iPad.
- Older generation iPads will allow a maximum of 11 pips.
- Newer generation iPads will allow a maximum of 17 pips.

Questions to Ask

- *How do you know that is the biggest pod that you can make.*
- *What will Benedict's pod look like? Describe it in detail.*
- *What will the shape of the pod look like and what colours are the pips?*
- *Does the pod have to be in this exact configuration or shape? Why?*
- *Do the colours matter?*
- *How do you know this pip (choose one) has to be this colour.*
- *What if Lucas added another pip, how would the pod change?*

Extending Student Learning

Early finishers can be encouraged to go further by asking them to draw:

- *What would happen if Lucas lifted a finger off the iPad?*
- *What if Lucas placed two more pip-fingers on the iPad?*

Assessment

Benedict had created the pips and pods that you can see in the photo. Draw a picture showing what the screen would look after Benedict places two more fingers on the right side of the screen.

TOUCH times

Skip Counting by Pips

Video Link

Summary

Students explore the effects of changing the number of pips on the composition of all of the pods using Grasplify. This more transformational approach to multiplication focuses on how each pip **spreads** across every pod.

Tasks

1. Demonstrate how to skip count by fives by adding more pods. Challenge students to find a way to skip count by five by changing the number of pips.

Skip Counting by Fives by Adding More Pods

TEACHER TIP

Skip counting by 5 by changing the number of pips requires students to first place 1 pip-finger and then create 5 pods for a product of 5. Students will then place down another pip-finger one at a time.

What to Watch For

- Ensure that students are skip counting by changing the pips. When first given this task, many pairs will skip count by adding more pods (pictured above), which is similar to repeated addition. Students are engaging in the process of **multi-plying** here, since they are seeing each new pod as a copy of the unit of pips.
- A hint might be to tell students to make 5 a different way than 5 pips and 1 pod.

Questions to Ask

- *What did you notice about Grasplify?*
- *How is skip counting by pips different than skip counting by pods?*

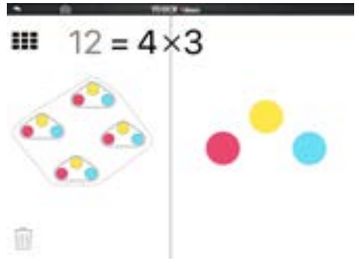
Extending Student Learning

- For pairs who have succeeded in skip counting by fives by changing the number of pips, you can invite them to try skip counting by 2 or 3 or 10.
- Invite students to skip count by a larger number that they are not as familiar with and to record the products in their notebooks.
- Make a product of twenty-five in Grasplify and skip count backwards by five by changing the number of pips.
- Make a product of fifteen in Grasplify and skip count backwards by three by changing the number of pips. Students will first need to decide how to make the product—3 pips, 5 times, or the other way around.
- Make a product of 24 in Grasplify and skip count backwards until you reach 12 by changing the number of pips. This can be done by counting backwards by twelves, starting with 2 pips and 12 pods; but it could also be done by counting backwards by sixes, starting with 4 pips and 6 pods.

Assessment

Sanna did some skip counting using Grasplify. She made this sequence of products: 3, 6, 9, 12, 15, 12, 9, 6, 3. Show or explain how she did this using drawings, words or a combination of both.

As a formative assessment task, project the image below.



Ask students to answer the following questions:

1. Sanna used Grasplify to skip count to 12. Describe each of the steps that she did if she was skip counting by changing the pips. If students need a hint, Sanna started with just one pip.
2. Imagine that Sanna placed another finger on the pip-side of the screen. Draw what we would see on her screen.

TEACHER TIP

It's also interesting to explore skip counting backwards. Watch that students don't do this by dragging pods to the trash, which emphasises repeated addition. Instead, encourage them to skip count backwards by changing the number of pips.

CAUTION!

The choice of the factors that go into the product matters for this task.

TOUCH times

Pips versus Pods

Summary

This task invites students to attend more closely to the different roles that the pips and the pods play in multiplication.

Video Link

Tasks

1. Students make 3 x 5 = 15 on their iPads. If your partner can only put one more finger on the screen where should it go to make the product the largest possible?

Begin with 3 x 5 = 15

Add a pip-finger or

Add a pod-finger

2. Challenge students to make 6 x 4 = 24. Where should the extra finger go to make the largest product?

Begin with 6 x 4 = 24

Add a pip-finger or

Add a pod-finger

3. Predict how to make the largest product for 5 x 5, 2 x 6, 6 x 2. Would we add another pip or another pod? After making their predictions, have students check to see if their predictions were correct.

4. Have students draw what 3 x 4 = 12 would look like in Grasplify using pencil crayons to show what the colours would look like in the pips and the pods. Then draw a second picture that shows what Grasplify would look like if another pip-finger was added. Finally, draw a third picture that shows what Grasplify would look like if you had 3 x 4 = 12 and created another pod.

What to Watch For

- We want students to notice the relationship between the pips and the pods and be able to explain how the product is influenced by the addition of a pip or a pod. By adding a pip to 3 × 5 = 15, it increases the product by 5 because an additional pip appears in each of the existing 5 pods. Alternately, by adding a pod, it increases the product only by 3 because an additional pod includes 3 pips.

Questions to Ask

- *What happens to the product when we add a pip-finger?*
- *What happens to the product when we add a pod-finger?*
- *Which creates the largest product? Why?*
- *Do you know if your prediction will be correct for sure, without trying it out on Grasplify? How do you know?*
- *What do you do when both numbers are the same? How do you know which will create the larger product?*

Extending Student Learning

- Early finishers can be challenged with a more open task, *Place as many pip-fingers as you want on the screen. Your partner will place as many pod-fingers as they want on the screen. Which side would you increase by one to get the largest product?* Students can continue to play this game by taking turns.

Assessment

Alternatively, invite students to come up with a context that models the pips versus pods situation they have explored. For example, if there are 4 motorcycles, each of which has 3 wheels, would you have more wheels in total by having an extra motorcycle or by having an extra wheel on each motorcycle? This might initiate a discussion about different circumstances in which it would be better to have more wheels or less wheels. A picture of a 3-wheeled motorcycle may help some students.

TEACHER TIP

Try to generate a more general statement, such as, "You can make a bigger product by putting the extra finger on the side with the smaller number."

Rearranging Pods

Summary

In this task students explore using the disembedding method to find different multiplication number sentence combinations. When using the disembedding method, students are breaking the whole into parts that are easier to manipulate or think about and then putting them back together into a whole to determine the product. For example, one way of disembedding 8 x 6 could be 8 x 5 = 40 and 8 x 1 = 8, which makes a product of 48.

Tasks

1. Project Grasplify onto a screen for the class to view. Tell students, ***I bought some cases of pop at the store. There are 6 in each case and I bought 5 cases.*** Using student suggestions, model what that would look like using Grasplify.
2. Say, ***I had 30 bottles but then my daughter asked me to buy two more cases because she wanted to try two other flavours. How can I use Grasplify to model or show this scenario? And how many bottles will I now have in all?*** Work through this as a class.
3. Students will use the disembedding method to create the following number sentences in Grasplify, using combinations of other number sentences that they already know. Have students record their number sentence combinations on mini whiteboard to keep track of how many different combinations that they discover.

   6 x 8

   9 x 13

   17 x 11

   13 x 21

What to Watch For

- Some students will want to remove the original 5 pods and make 2 pods instead. Create 6 x 5 for students to see. Remove your fingers to reset the screen and then create 6 x 2. Comment that, These are easy products to find, 30 and 12.
- From 6 x 5, others may suggest placing 2 additional pods on the screen. Encourage them to set the 2 new pods slightly apart from the 5 existing ones.

Video Link

SETTINGS

For this task, make sure that the Numeric Factor Values and the Numeric Product Values are turned off in settings.

TEACHER TIP

Make sure that students group the sets of pods that they create and that they take a screenshot of their solution.

TEACHER TIP

Students can turn the Numeric Product Values back on to verify their solutions.

Questions to Ask

- *How would you describe this as a multiplication equation?*
- *So, can I figure out 6 x 7 by first finding 6 x 5 and then finding 6 x 2? What do you think? When we break multiplication equations apart like this, it's called disembedding.*
- *Are there other ways that we can figure out 6 x 7 by using this disembedding method?*
- Encourage them to discuss why they like certain disembeddings more than others.

Extending Student Learning

- Once pairs who have successfully found different number sentence combinations for each of the given number sentences, challenge them to figure out the product for each.
- Early finishers can be invited to make up a set of factors of their own choosing.

Assessment

Tasha took this screenshot. Write down the multiplication equation and explain how she used disembedding to find the product.

Multi-plying Factors

Video Link

VOCABULARY

The vertical lightning bolts go up and down.

The horizontal lightning bolts go side to side.

TEACHER TIP

The product is the result of the intersection points created by the horizontal and vertical lightning bolts.

Summary

In Zaplify, all numbers represent magnitudes and the unit of the product derive from the multi-plying of the factors. Factors are represented by the number of lightning bolts and the product is represented by the intersection points. The covariation between the factors and how it expresses the product is explored in this activity. For example, in the photo to the left, the product "ten" is established at the intersection of the horizontal and vertical lightning bolts.

Tasks:

1. Using Zaplify, ask students to:
   - Make a horizontal line.
   - Make two horizontal lines.
   - While holding two horizontal lines, make a vertical line.
   - Make three vertical lines.
   - What is the product?
   - What are the factors making the product?
2. Have students figure out how to create a product of ten using Zaplify.
3. Have students create a product of one using Zaplify.

What to Watch For

- It is common for students to initially place ten fingers down to create a product of ten.
- If this strategy persists, ask students, *What is the product when you pressed ten fingers down? How could you make the product ten?*

Questions to Ask

- *How did you figure out how to create a product of ten? Did you get this right away? If not, what did you do first? How did you know that was incorrect?*
- *Were you surprised that you needed two lines to make a product of one?*
- *What does the second line do to the first line? What does it create?*
- *Where is the point? When did it appear? If I want to make a single point right here* (point to a location on the screen that is different from where the previous lines intersected), *how should I place my fingers?*
- *Does the order of the lines matter? Why or why not?* Make explicit that the number of lines represent the factors, which are the black numerals on the screen, and the number of orange intersection points represent the product, which is the orange number at the top of the screen. The ideas represented by the lines and the intersection point in this case are important ideas for thinking multiplicatively.

Extending Student Learning

- Encourage students to try making 12, try making 15, try making 1.
- As pairs successfully create a product of one, direct their attention to the relationship between the factors and the product, by asking, *You made one with two fingers here. Can you make a single point right here?* Point to a location on the screen that is different from where the previous lines intersected. Some students will always place their fingers in the same order, always creating horizontal (or vertical lines) first. Invite children to explore how the order of the orientation of the lines influences the product.

Assessment

If you press your fingers like the ones in the picture, draw what you would see in Zaplify. Include as many details as possible and write the multiplication sentence that would go with this situation.

TEACHER TIP

Students may think that a point can be created only at a specific location. The goal is for students to figure out that a point can be created at any location as long as they create an intersection at that point.

TEACHER TIP

We want students to notice that in order to create an intersection point, they need two types of lines (vertical and horizontal). The number of intersection points is the product.

Video Link

TEACHER TIP

We want students to understand that the product of two factors will be the same, regardless of their horizontal or vertical placement.

Order Doesn't Matter - Part A

Summary

In Zaplify, the order of the factors doesn't matter and students can focus on how the product of two numbers is the same, whether they are made with 3 horizontal lines and 2 vertical ones or the other way around. In this task, students explore the commutative property of multiplication by creating the same product in two different ways and by comparing the shapes of the products.

Tasks

1. Say, *One partner will place two fingers along the left side and three on the bottom to produce six. The other partner will make a sketch of what Zaplify is showing.* [Alternatively, students could take a screen shot.] *Now, place three fingers along the left side and two fingers on the bottom.*

2. Make the product 6 by pressing six fingers along the side and one finger along the bottom and project this on the board. Ask students to compare the three shapes.

What to Watch For

- Some students may only focus on the products. If this happens, prompt students to compare the general shape of the lines or the number of horizontal lines to the number of vertical lines sketches.

Questions to Ask

- *What is the new product? How does it compare to the sketch that you made?*

- *How many horizontal lines are in this sketch* (point to their first sketch) *and how many vertical lines are in this sketch* (point to their second sketch)*?*

- *Which sketch seems different than the other two? Why is it different than the others?*

- *Do you see any connection between the first and the second sketch?*

- *What should I do to the second sketch so that it would look exactly like the first sketch?* **This question would prompt students to rotate their sketches.**

- *When we rotate the sketch, we change the orientation of the lines. Can you come up with a multiplication rule about the order of the factors?*

Extending Student Learning

Early finishers can be asked to make another product in two different ways. When students are asked such open-ended questions, some of them may not create the products based on the commutativity property. For example, they can produce 12 either as 3 x 4 or as 2 x 6. If this happens, direct them back to their drawings and ask them to compare how they produced 6 in two different ways and how they produced 12 in two different ways. Prompt them to come up with a rule to make their drawings the same. This might prompt them to rotate one of their sketches.

Assessment

The desks in Selin's classroom are organised in rows like the picture below. There are four rows of desks organised this way. One day Selin's class had to move to another room. However, the new room was smaller and could only fit four desks in each row. How many rows of desks will be in the new room? Draw how you can model this situation in Zaplify.

Order Doesn't Matter – Part B

Video Link

Summary

Students continue to explore the idea of commutativity. Mathematical concepts frequently have multiple conceptualisations, each of which can be more suited to particular contexts—this is an important part of mathematics that may help students move away from the assumption that there is only one right answer or one right way of solving a problem.

Task

1. Explain to students that Claudia used ten fingers to produce twenty-one. Challenge them to show two different ways she could have done this and to take screenshots of their findings.
2. Have students make the product 24 by pressing 6 fingers along the left side and 4 fingers along the bottom. Then ask them find other ways to make twenty-four.

What to Watch For

- Students are given a restriction in this task (they must use exactly ten fingers). This restriction limits the answer to two solutions (3 x 7 and 7 x 3). Otherwise, students might come up with 21 x 1 and 3 x 7 as two ways to make 21 and this would prevent them from exploring commutativity.
- Some students may have difficulty getting started with this task. If that happens, ask them how they would produce 25. Once they have created 5 x 5, prompt them to change the product by changing the location of their fingers until they make the product 21.

TEACHER TIP

Shifting between the two photos on the screen might be difficult for students to compare the shapes. If the iPad is a newer version, you can display two photos at the same time on the screen. Otherwise, invite two students to place their iPads side-by-side to compare the images.

Questions to Ask

- *Did you get this right away? If not, what did you do first? How did you know that was incorrect?*
- *How did you decide which factors to use to make the product twenty-one? How did it help you to figure out another way to make the product twenty-one?*
- *How does the shape of the first product compare to the shape of the second product?*
- *How many horizontal lines are in the first photo and how many vertical lines are in the second photo?*

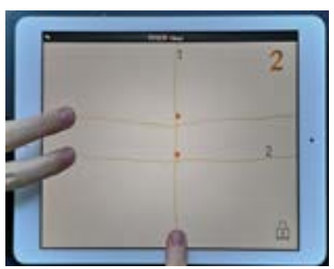
Extending Student Learning

Early finishers can be asked to find as many ways as they can to make 24, draw sketches of each and identify which sketches look the same.

Unitising

Summary

An important part of Zaplify involves seeing how the intersecting bolts of lightning work. For example, in the pictures below, two horizontal lines are established, and with the addition of one vertical line, a two-ple (two points on each vertical line) is created at the intersection points. Making two points at once highlights the process of **unitising** (two becoming one), which doesn't typically come out in traditional ways of talking about multiplication.



Video Link

Task

1. Project Zaplify onto a screen for the class to view. Make one point and explain that 1 point on a single line is called a one-ple. Then make two points as in the left photo above and state that if we call a line with one point a one-ple, then a line with two points would be called a two-ple. Demonstrate making two points in a different way, as shown in the photo above on the right. Challenge students to figure out different ways to make three points on a single line.

What to Watch For

- It is common for students to initially place only three fingers down on either the side, or on the bottom.
- Students may initially make one point and then press two more fingers. If this strategy persists, tell students, *You should make three points. Three points should appear at the same time.*
- The goal of this task is for students to develop fluency in performing the **unitising** action with different numbers, using only one multi-ple making finger.

Questions to Ask

- *Did you get this right away? If not, what did you do first? Can you explain how you knew that was incorrect?*
- *How did you figure out how to create a three-ple instead of three one-ples?*
- *Did you find a different way of doing this?* This provides an opportunity to discuss and demonstrate how 3 x 1 = 3 and 3 = 1 x 3 are both ways of creating a three-ple.
- *Does the order of the lines in this case matter? Why or why not?* Make explicit that the black numerals represent the factors and the orange numeral(s)represents the product.
- *What will happen if we add another vertical line? What happened to the horizontal line? What do you notice about the number of points?*
- While the pair holds their fingers on the screen, ask, *What will happen if we add another horizontal line?*
- *What happened to the horizontal line?*
- *What do you notice about the number of points?*
- *What should we call this?*
- *What would a five-ple or a six-ple look like?*

Extending Student Learning

Ask early finishers how they can make four points on one line. Seven points? I wonder how many points can you create on a single line?

Assessment

1. Draw how the screen would change if you:
   - place one finger along the bottom
   - place two fingers along the bottom
   - place one finger along the bottom and one finger on the left side.
2. Show how to make 6 intersections in Zaplify in two different ways.

TEACHER TIP

When there is only one three-ple on the screen, students might equate the product with the multi-ple as a generalisation.

When both a vertical and a horizontal line are added, there will be two four-ples on the screen. Describing this as two four-ples, students can identify the difference between a multi-ple and a product.

TOUCH times

Skip Counting - Part A

Video Link

Summary

Students focus on how changing the number of lines affects the composition of all the perpendicular lines. This is a more transformational approach to multiplication than repeated addition and focuses on how the intersection points are spreading across every perpendicular line in both directions.

Task

1. Project Zaplify onto a screen for the class to view. Create two horizontal lines and model skip counting by two by placing one finger at a time on the screen to create vertical lines. Point out to students that the product is changing. Students will likely recognise this as skip counting. Challenge students to skip count by five by changing the number of horizontal lines.

WHAT TO WATCH FOR

Ensure that students are skip counting by adding horizontal lines. We are intentionally holding the number of vertical lines constant and seeing what happens by adding more horizontal lines.

2. Starting with a product of twenty-four, ask students to figure out how to skip count backwards by only changing the horizontal lines so that they end up with a product of twelve. This can be done in several ways. Student can skip count backwards by:

Skip Counting by:	Start With:		Skip Counting Pattern
	Horizontal Lines	Vertical Lines	
12	2	12	24-12
6	4	6	24-18-12
4	6	4	24-20-16-12
3	8	3	24-21-18-15-12
2	12	2	24-22-20-18-16-14-12

What to Watch For

- Skip counting by 5 by changing the number of horizontal lines will require students to place 5 fingers on the bottom of the screen and 1 finger on the side of the screen. This creates 5 points on the horizontal line, and by placing one more horizontal-finger at a time, Zaplify will skip count by 5.
- If students persist with skip counting by changing the vertical lines, ask them to make 5 in a different way. This may prompt them to start by making five vertical lines and one horizontal line.
- You may also want to ask students to record the successive products that they discover in their notebooks. They could also draw pictures to show what each product looks like in Zaplify.

Questions to Ask

- *How can you skip count up to 25 by five-ples?*
- *How will the product will change when you lift (or remove) a finger?*
- *How could you skip count backwards by 5?*
- *How did you figure out how to make the product twenty-four? Which factors did you use? Did you get this right away? If not, what did you do first? How did you know that was incorrect?*
- *How did you decide which lines to use for counting backwards?*
- *What is the name of that m-ple that you were lifting?*

Extending Student Learning

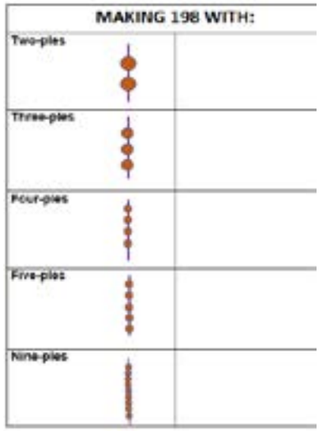
- As you walk around and see that some students have succeeded in skip counting by fives by changing the number of horizontal lines, you can invite them to try skip counting by 3, 7 or 10. You might also invite the students to skip count by a larger number that they are not as familiar with, and to record the products in their notebooks
- Challenge early finishers to make a product of fifteen and skip count backwards by three by only changing the number of horizontal lines. Students will first need to decide how to make the product - 3 vertical lines, 5 horizontal lines, not the other way around.

Assessment

Project the image to the right and ask students to complete the following:

1. Sanna used Zaplify to skip count to twelve. Describe each of the steps that she did if she was skip counting by changing the horizontal lines. If students need a hint, Sanna started with just four vertical lines.
2. Imagine that Sanna placed another finger on the bottom of the screen. Draw what we would see on her screen.

Video Link



Skip Counting - Part B

Summary

Students continue to investigate the idea of spreading, which is more multiplicative in nature than repeated addition.

Task

1. Ask students to determine if it is possible to count backwards from twenty-four by using five-ples.
2. Challenge students to make the product 198 by counting up with the multi-ples on this chart (two-ples, three-ples, four-ples, five-ples and nine-ples). Only three of these multi-ples will work for skip counting to 198. Students need to figure out which three will work.

What to Watch For

- Students can create multi-ples either on horizontal lines or on vertical lines.
- Students might tap very quickly to increase the product. Prompt them to slow down as the product becomes larger.

Questions to Ask

- Are there any that you know immediately will not work? How do you know?
- *Can you reach 198 with two-ples?*
- *Which other multi-ples did you use to make the product 198?*
- *Can you reach at 198 using five-ples? Why not?*
- *What was the product before your final tap? How many more points do you need? Can you create that many points with one touch? How many points was your finger creating with one touch?*
- *Did you notice a pattern while skip counting with five-ples? W*hen skip counting by five-ples, the ones digit will always be either zero or five.
- *How does this help me know immediately if I can make 198 with five-ples? What about ten-ples? What about two-ples?*

Extending Student Learning

- Additional multi-ples (six-ples, seven-ples, eight-ples, ten-ples) can be given to early finishers.

Video Link

Doubling

Summary

Students create pictorial representations of multiplication using *TouchTimes* to investigate the mathematical strategy of doubling. In this task students explore the idea of simultaneous multi-plying, simultaneously creating a double of an original quantity.

Task

1. Ask students to double a product of three so that it is six. Once they've done that, they can double the product again to make twelve. Then double it again to make twenty-four. Write the 3, 6, 12, 24 sequence on the board for students to refer to while working.

Questions to Ask

- *Can you double the product of three in a different way?*
- *What was happening that caused the product to double?*
- *How do the factors change when you double the product?*

Extending Student Learning

- Challenge early finishers to start with a product of five and double it.
- Another extension is to invite students to triple the product. They will quickly run out of fingers, so this might also be a task where they imagine how to triple the product and record their thinking on paper.

Assessment

Show or explain how you would solve this problem using Zaplify:

There are six lanes in a swimming pool. In Mathew's swimming lesson, every lane is shared by two students. When another class joins Mathew's lesson, the number of students that share a lane is doubled. How many students are swimming in the pool now?

Halving

Summary

Students create pictorial representations of multiplication using *TouchTimes* to investigate the mathematical strategy of halving. In this task students explore the idea of multi-plying, simultaneously creating half of an original quantity.

Video Link

Task

1. Challenge students to start by making a product of twenty and then figure out how to halve the product so that it is ten. Once done that, halve the product again to make five. Write the 20, 10, 5 sequence on the board for students to refer to while working.

What to Watch For

- Students will have to shift from adding more fingers to removing half of their horizontal fingers each time.

- Some students might lift their fingers one by one. If that happens, prompt them to lift their fingers all at once. Encourage them to decide how many fingers they will lift before lifting any fingers. For example, if the student creates twenty, as in the photo above, you can point to the horizontal lines and ask, *How many fingers do you need to lift to halve this factor? Let's see how this changes the product.*

Questions to Ask

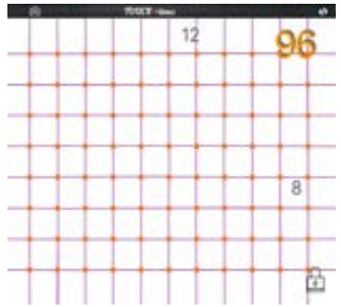
- *How is doubling different from halving?*
- If a student creates 20 on Zaplify by making 4 horizontal lightning bolts, ask them, *What do you have to do next in order to halve the product so that it becomes ten? Which side do you lift your fingers from? How do you know that?*
- If a student creates 20 on Zaplify by making 5 horizontal lightning bolts, ask them, *Why can't I halve twenty by changing the number of horizontal lines? Is there another way to halve twenty?*

Extending Student Learning

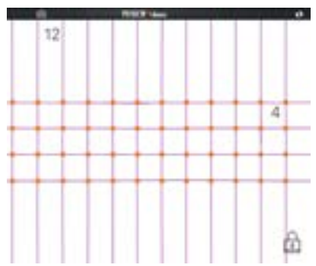
- As pairs successfully complete the given task, challenge them to find other ways to halve the product. This will result in the impossible task of halving the product of 20 by halving the factor of five.
- Early finishers can be challenged to try halving the product of 40 in two different ways.
- Students can also be asked to repeatedly halve the product starting from 50, which would quickly produce a challenge (halving 25).
- Be sure to try a challenging question like 3 × 7 and ask about halving.

Assessment

1. If you know that 48 x 12 = 576, how would you use that information to find the answer to:
   - 48 x 6
   - 24 x 12



2. How would you find the product in the pictures below by using picture above?



TEACHER TIP

Challenging questions direct student attention to reunitising (for 21 we would need 10.5 units of 2).

Maximising the Product – Part A

Video Link

Summary

Students investigate the covarying relationship between each factor and the product by changing the factors and observing how each factor changes the product differently. By comparing how the factors interact to increase or decrease the product, students try to determine how to maximise the product.

Task

1. Have students make a product of six using the factors two and three and then predict where to place one more finger on the screen to maximise the product. Encourage them to predict what the new product will be. Then students try it out. If their prediction was incorrect, encourage them to figure out why. If their prediction was right, how did it work?

2. While projecting 3 x 3 on the screen, ask students where to place one finger to maximise the product. Ensure that students understand that for square numbers it doesn't matter whether they increase the horizontal lines or the vertical lines by one to maximise the product.

A Square Number

1 More Vertical Lightning Bolt

1 More Horizontal Lightning Bolt

What to Watch For

- Remind pairs to make their predictions first and then try it out on Zaplify.
- Some students may lift their fingers as soon as they place them on the screen. Prompt them to hold their fingers on the screen until the end of the task.
- Once students have created 2 x 3 = 6, if they add multiple fingers to maximise the product, prompt them to start over, reminding them that they can only use only one finger in this task.

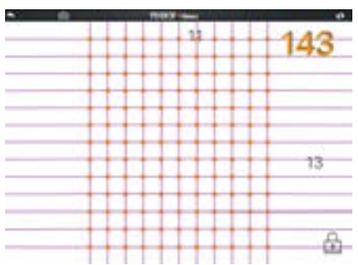
Questions to Ask

- *What happened when you made a horizontal line?How many new points appeared on each vertical line? How many new points in total appeared? Is there another horizontal line with the same number of points?* In order to draw attention to the multi-plying aspects of multiplication, ensure that children explain what happened on the vertical lines when another horizontal line was created.
- *Do you notice anything about the three-ples?* Draw attention to the number of points on each horizontal line and the relationship between the points and the number of vertical lines.
- *When you increased this factor (point to the horizontal lines) to maximise the product, what was happening that caused the product to increase more?*

Extending Student Learning

- Early finishers can be challenged with a more open task, *Place as many fingers as you want on one side of the screen. Your partner will place as many fingers as they want on the other side of the screen. Which factor should you increase by one to maximise the product?* Students can continue to play this game by taking turns.
- You can extend this task by asking students, *Place two fingers along the side and three along the bottom to produce six. Your partner can put two more fingers on the screen. Where should they be placed to maximise the product? Where should they be placed to minimise the product?*
- Ask students to find as many products as they can and to record these products.

Assessment



1. How will the product change if you increase the first factor from 13 to 14?
2. How will the product change if you increase the second factor from 11 to 12?

Explain your answers by drawing on the picture of 13 x 11= 143 above.

Maximising the Product – Part B

Video Link

Summary

Students continue to create square numbers. This task is an arithmetic version of what students will later see in algebra, when they work with the product of binomials.

Task

1. Have students place two fingers along the left side and two fingers along the bottom to produce four and then increase each factor by one. Have them draw a sketch of what they see. Then use Zaplify again to make 3 x 3 and increase each factor by one. Have them draw another sketch of what they see. Have them predict how many new points will appear if they increase each factor by one in 4 x 4? Then have them draw a picture showing this prediction.

What to Watch For

- Prompt students to work as a pair by taking turns. While one student holds their fingers and increases the product, the other student draws a sketch. Prompt students to use pencil crayons to depict the intersection points.
- Some students might add lines to an existing sketch. If that happens, prompt them to make a separate sketch for each situation and compare all of them at the end. They can make their drawing on the iPad, using the built-in drawing app—they might even want to start with a screen image of, say, 3 x 3, and simply add on the required elements.

Questions to Ask

- *When you added a finger, how many more intersection points did this create?*
- *Do you see a relationship between the number of points that are created by each finger?*
- *The second finger creates one more point than the first finger does. What do you think caused this additional point?* Draw students' attention to the intersections between the lines.

Extending Student Learning

- Early finishers can be asked to add more than one finger to the factors and to predict what would happen by making a quick sketch.

TEACHER TIP

We want students to notice that there's an extra orange point that must be counted, in addition to the horizontal and vertical lines, which is the result of the covarying of the factors.

Maximising the Product – Part C

Video Link

Summary

Students continue to maximise the product. This task poses a restriction on the number of fingers to direct students' attention to the covarying relationship between the factors and the product.

Task

1. Have students find out if it is possible to make a product larger than twenty-five using exactly ten fingers.

What to Watch For

- Some students might have difficulty starting the task. If this happens, prompt them to make a product they are familiar with (perhaps 3 x 4 = 12) and then change the factors to try to get a bigger product. Ask them to try different combinations by changing the locations of their fingers.
- Some students might use less than ten fingers. If that happens, prompt them to press exactly ten fingers down.
- Some students might aim to produce 10 by using less than ten fingers. If that happens, explain that the total number of fingers should be 10, not the product.

Questions to Ask

- *What is the smallest product? With which factors did you produce it?*
- *What is the largest product? Which factors did you use?*
- *Do you see any connection between the factors and the size of the product?*
- *How does the difference between the factors change as the product increases?*

Extending Student Learning

- Early finishers can be asked to make the largest product using only 11 or 12 fingers. They might also try to predict the largest products for larger numbers, without using Zaplify.

Assessment Ideas

Drawing

Students can be asked to show their understanding of many tasks through drawing. Depending on the task, a single drawing may be sufficient, or students can be provided with a comic strip panel in order to encourage a sequence of drawings that demonstrate their understanding. An example prompt that could be used for the Doubling task in either Grasplify or Zaplify could look like this:

Make a comic strip drawing that shows what the Grasplify (or Zaplify) screen would look like if you start with 1 x 4 = 4. Show how it would change if you double the product to 8 by only changing the number of pips (or horizontal lines). Be sure to use your pencil crayons to show the colours. If you would like to continue doubling the product using pips (or horizontal lines), you can add more frames to your comic strip.

Another example of using drawing for assessment can be found on page 35 for the Pips vs Pods task.

Written Response

Students can demonstrate their understanding by providing written instructions to someone who has never used *TouchTimes* before, about how to complete one of the tasks. This assessment idea can easily be adjusted for either Grasplify or Zaplify. An example prompt that could be used for the Halving task could look like this:

Provide step-by-step instructions for someone who has never used Grasplify (or Zaplify) before.

1) Explain how to create 12 = 3 x 4. Don't forget to use the words pips and pods (or vertical and horizontal) in your explanation. You can use RS for the right screen and LS for the left screen.

2) Explain how to halve the product by only changing the pips (or the vertical lines).

Audio and Screen Recording Apps

Using an audio and screen recording app, such as Show Me, students demonstrate and explain their understanding. An example prompt that could be used for Many-to-One (Part A) and Skip Counting by Pips could look like this:

Using the Show Me app, demonstrate and explain to your parents how to create a 4-pod and then skip count by pods. Once you've done that, demonstrate and explain how you can also skip count by fours in Grasplify using only the pips.

More Assessment Ideas

Screen Shots

Screen shots of *TouchTimes* can be used to prompt student responses that can then be used for assessment. Student responses can include drawings, written or verbal explanations. Students can be asked to explain what is happening in the screen shot or can also be promptd with "what would happen if..." scenarios. Example prompts could look like this:

Left photo: Katie creates 5 x 3 = 15 using Grasplify, and it looks like this. Draw what would happen if Katie lifted a finger off the iPad or draw what would happen if Katie placed two more pip-making fingers on the left screen.

Right Photo: Katie creates 5 x 3 = 15 using Zaplify, and it looks like this. Draw what would happen if Katie lifted a finger off of the side or draw what would happen if Katie placed two more fingers on the bottom.

Word Problems

Students can be given word problems and asked to show how these using either Grasplify or Zaplify.

A couple examples of word problems that could be used are:

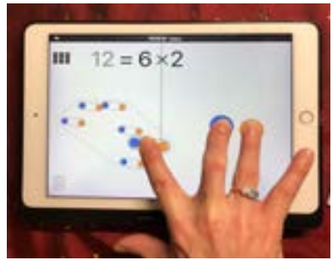
Some kids were playing a team game. Each team has 3 kids and there are 4 teams. Show this using Grasplify.

I was in a classroom where desks were organized into rows. Each row had 6 desks. One classroom has enough space for 5 rows of desks. How many desks would be in 2 classrooms? Use Zaplify to show your answer.

Additional Task Ideas

Grasplify

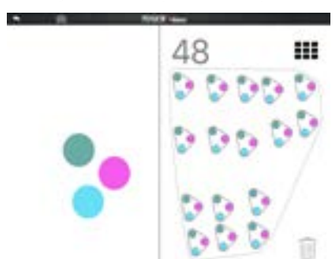
Task: Make 3 x 5 = 15. What do you think will happen if you move one pip-making finger to the other side of the screen? Will the product be bigger or smaller?



Task: Make 6 x 4 = 24. What do you think will happen if you move one pip-making finger to the other side of the screen? Will the product be bigger or smaller?

Class Discussion: Have students predict what will happen with other examples: 5 x 5, 2 x 6, 6 x 2. Lead the group to generate a more general statement, such as “the product will increase if the number of pips is smaller than the number of pods” and try to explain why that happens and what happens when the number of pips and pods are the same.

Task: (With numeric factor values setting turned off) In the image below, Kumari made 3 x 16, arranging the pods so that she could see 10 of them towards the top of the screen and 6 towards the bottom. That helped her figure out that 3 x 16 was actually just 3 x 10 added to 3 x 6. Use the same strategy to find the following products: 5 x 14, 4 x 23, 6 x 15, 3 x 29 and take a screen shot of how you decomposed the pod.



Task: How can you make the largest product with only 11 or 12 fingers?

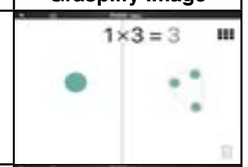
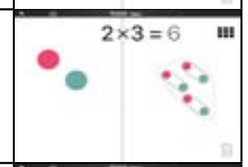
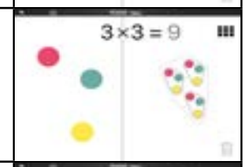
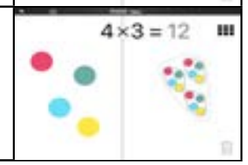
Class Discussion: Ask students what they’ve discovered about maximising the product. You are looking for descriptions such as ‘the product is biggest when the difference between the number of pips and the number of pods is smallest’.

For more exploration, ask students whether it is possible to use less than 10 fingers to make a product that is bigger than the ones they made with exactly 10 fingers. Children could be invited up to the front to try these out. For example, 4 x 4 = 16 only uses eight fingers, but it is greater than 1 x 9 = 9, which uses 10 fingers.

Task: Make 5 x 5 = 25 and press the array button. Make a sketch of what is displayed on the iPad. Now make two other products that use exactly 10 fingers and draw what you see displayed. Compare the drawings. How are they the same? How are they different?

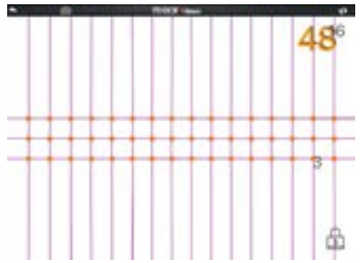
Ensure that student drawings are detailed enough to show the dimensions and size of each array. The goals here are to introduce the array button and draw attention to the shape of the array and that different products produce different arrays/shapes.

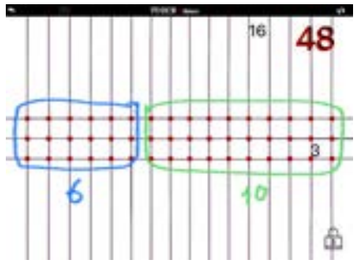
Task: Students explore changing the size of units (the multiplicand). Using a table highlights the functional view to multiplication—here the function is to triple. The table below shows that the relation between the number of pips (1), the product (3) and the number of pods is 3. This table can be extended by changing the number of pips. The number of pips has increased by 1 in each row. The functional relation is being emphasised because it shows that if 1 pip tripled makes 3, then 4 pips tripled makes 12.

Number of Pips	Product	Grasplify Image
1	3	
2	6	
3	9	
4	12	

Zaplify

Task: In the image below, Kumari made 3 x 16 and decomposed the product into two parts by drawing blue and green boxes. That helped her figure out that 3 x 16 was actually just 3 x 10 added to 3 x 6. Use the same strategy to find the following products: 5 x 14, 4 x 23, 6 x 15, 3 x 29 and take a screen shot and decompose them by drawing boxes.





Task: There are 3 wheels on a tricycle. How many total wheels are there if there are 4 tricycles? This problem can be modelled in Zaplify by making 3 horizontal lines for the wheels and a vertical line for each tricycle. Tricycles can easily be changed to bicycles (change 3 to 2), or cars (change 3 to 4). Zaplify offers a more dynamic and fluent process to draw attention to the **spread** of horizontal lines across the vertical lines (or vice versa).

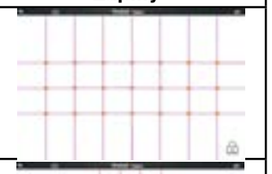
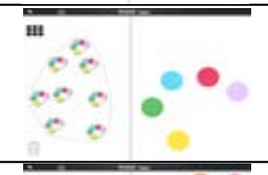
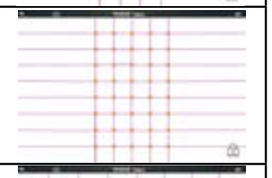
Tables can be used to highlight the functional view of multiplication. The first example shows the relation between the number of horizontal lines (1), which has three points on it (three-ple), the product (3) and the number of vertical lines (3). The number of horizontal lines has increased by 1 in each row. The functional relation is being emphasised because it shows that if 1 three-ple makes 3, then 4 three-ples makes 12.

Number of Three-ples	Product	*TouchTimes* Image
1	3	
2	6	
3	9	

Grasplify or Zaplify

The tasks included in this section can be used effectively with either Grasplify or Zaplify.

Task: (off-line) The equation is missing. Write down what equation should have appeared in *TouchTimes*.

Equation	Grasplify	Zaplify
Grasplify: 3 x 7 = 21 Zaplify: 3 x 7 = 21 or 7 x 3 = 21 21 = 3 x 7 or 21 = 7 x 3		
Grasplify: 4 x 7 = 28 Zaplify: 4 x 7 = 28 or 7 x 4 = 28 28 = 4 x 7 or 28 = 7 x 4		
Grasplify: 5 x 8 = 40 Zaplify: 5 x 8 = 40 or 8 x 5 = 40 40 = 5 x 8 or 40 = 8 x 5		
Grasplify: 7 = 1 x 7 Zaplify: 1 x 7 = 7 or 7 x 1 = 7 7 = 1 x 7 or 7 = 7 x 1		

Task: Show how you would solve these problems using *TouchTimes*:

(1) A store sells crayons in packages of 6. You decide to buy 3 packages. How many crayons will you have?

(2) You watched 4 seasons of a show and each season had 7 episodes. How many episodes did you watch in all?

(3) A school has 20 clocks. They came in packages of 4. How many packages were bought?

(4) There are 36 kids that have been organised into 9 groups. How many kids are in each group?

(5) There are 24 students in a class. Their teacher wants to arrange their desks into equal groups. What arrangements can be made so that each group has the same number of desks in it?

Task: Students work in partners. Partner 1 creates a multiplication expression (in Grasplify using pips and pods or lighning bolts in Zaplify) that Partner 2 is unable to see. Partner 1 then describes what they have created to Partner 2, who must recreate it on their own iPad.

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