Doubling and Halving

PART A

Introduction (for teacher's reference only):

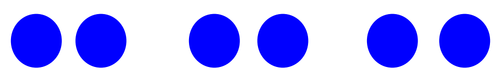
Imagine that you are teaching 6th graders and ask the following question:

> I know that the product of 18 times 23 is 414.
> If I increase the first factor from 18 to 19, what would be my new product?

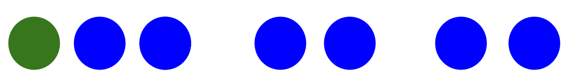
Canan immediately answers "415", which is incorrect. Why do you think Canan is thinking like this? How do you think the rest of your students would answer this question?

Research shows that the majority of the students would think like Canan. One reason for the widespread incorrect answer to these types of questions might be that students are thinking additively instead of multiplicatively. When students think additively, they see the multiplicative factors as independent numbers which represent the same type of object. Accordingly, they think that changing one factor only affects one object, where the other factor is insignificant.

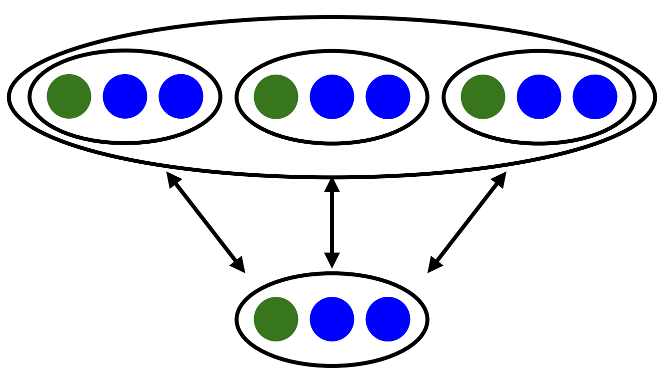
For example, when you start with 2 x 3, as represented below:



If the first factor 2 is increased to 3, students who are thinking additively may be thinking of <u>only</u> adding one more (green) object to get 7 objects.



While students who are thinking multiplicatively are likely to be thinking of adding one more (green) object into <u>each</u> of the 3 groups to get 9 objects.



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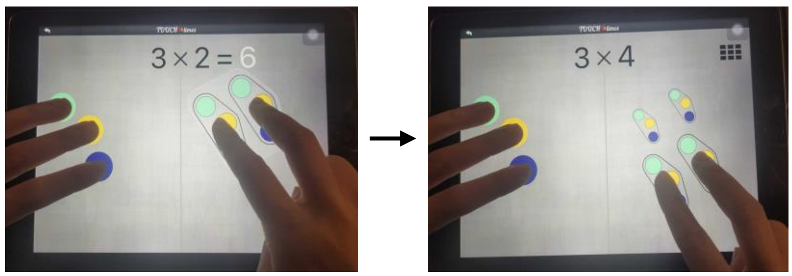
Thus, to help students develop multiplicative thinking, we propose an important idea of multiplication, namely, ***multi-plying*** where multiplication is seen as an action that simultaneously creates multiple versions of an original unit. This is similar to carbon paper copying where multiple copies of an original document can be created all at once.

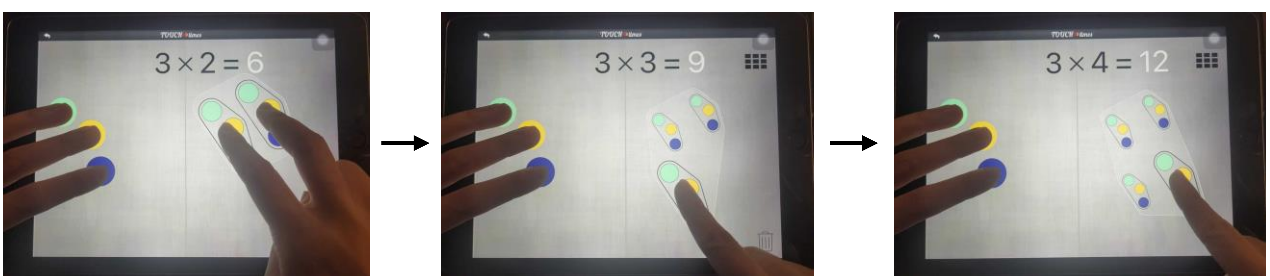
In Grasplify, when students press or lift multiple fingers all-at-once, they would be able to observe how the original unit is replicated multiple times all-at-one. For example, starting from 3 x 2 = 6, students can press two fingers all-at-once to get 3 x 4 =12.



Simultaneous replication of copies

This contrasts with the idea of multiplication as repeated addition or the action of sequentially adding copies. This is similar to copying machines where multiple copies of an original document can be created one at a time.

In Grasplify, when students press or lift one finger at a time, they would be able to observe how the original unit is replicated one at a time. For example, starting from 3 x 2 = 6, students can press one finger to get 3 x 3 =9 and press one finger again to get 3 x 4 = 12.



Sequential replication of copies

Specifically, with this task, students can experience this idea of *multi-plying* through exploring what happens when we double and halve quantities.

Doubling and Halving

Students will start by exploring how we can double the product – do we double both factors or do we only double one factor? After which, we will also explore how we can halve the product. This will differ slightly from doubling as not all factors can be divided by two exactly, i.e. the odd factors. Thus, students will also explore the restriction on the factors when trying to halve a product. For example, consider 14 x 5 = 70:

When we double the factor of 14 to 28, we get 28 x 5 = 140, so the product is also doubled. When we double the factor of 5 to 10, we get 14 x 10 =140, which also doubles the product.

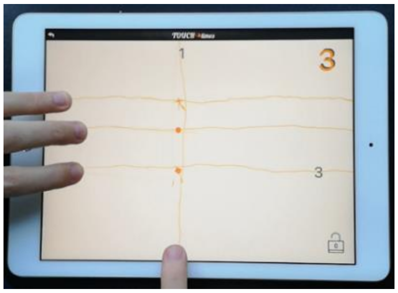
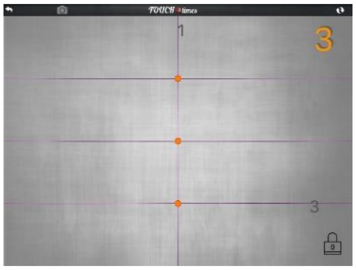
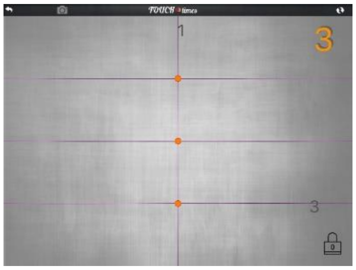
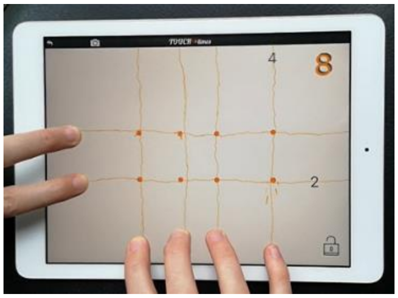
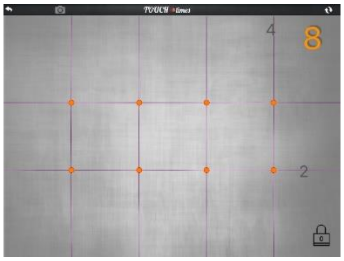
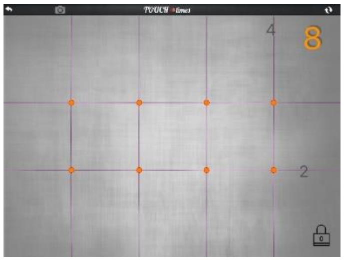
But when we double both the factor of 14 to 28 and 5 to 10, we get 28 x 10 which is four times the original product instead of double.

Similarly, when we halve the factor of 14 to 7, we get 7 x 5 = 35, so the product is also halved. But we are not able to halve the factor of 5 (without getting into decimal numbers).

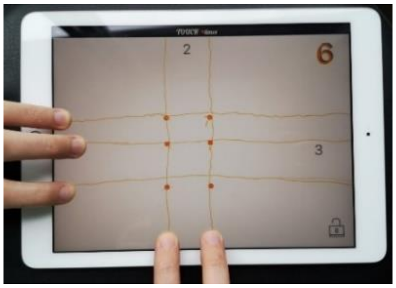
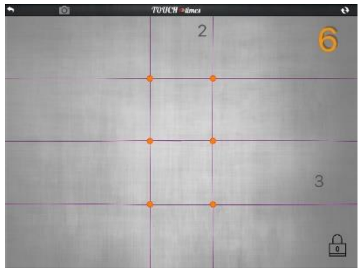
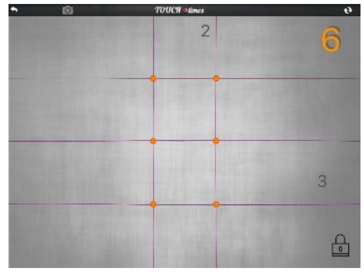
Doubling and Halving

Task: Doubling and Halving

Part 1 - In this part of the task, you will double the three products given below by touching your fingers to the iPad. First, make the products as shown in the photos. Then double the products in two different ways, draw what you saw on the screen, and describe where you placed your fingers (see example below). In the box at the bottom, write the corresponding equation.

Original Product	Draw Double the Product (1st way)	Draw Double the Product (2nd way)
		
I placed 1 finger on the bottom and 3 on the side.	I *doubled* my finger on the bottom.	
$1 \times 3 = 3$		
What do you notice?		
Original Product	Draw Double the Product (1st way)	Draw Double the Product (2nd way)
		
I placed 4 fingers on the bottom and 2 fingers on the side.		
$4 \times 2 = 8$		
What do you notice?		

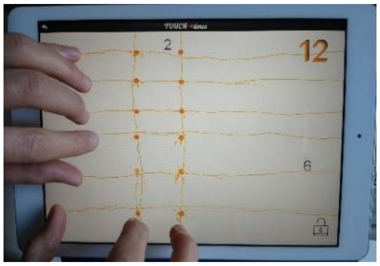
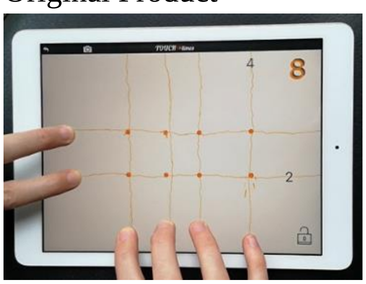
Doubling and Halving

Original Product	Draw Double the Product (1^{st} way)	Draw Double the Product (2^{nd} way)
		
I placed 2 fingers on the bottom and 2 fingers on the side.		
2 x 2 = 4		
What do you notice?		

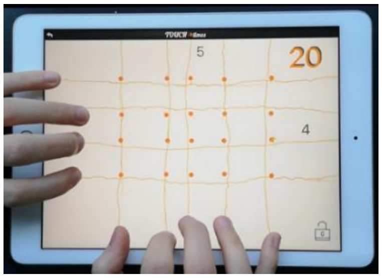
Sam makes the product 56 by using the factors 14 and 4. They want to double 56 and thinks that one way to do this is doubling both 14 and 4 at the same time. What do you think about Sam's doubling strategy? Explain. If you do not agree with the strategy, what could Sam do instead?

Doubling and Halving

Part 2 – In this part of the task, you will halve the three products given below by lifting your fingers from the iPad. First, make the products as shown in the photos. Then halve the products in two different ways (if possible), draw what you saw on the screen, and describe which fingers you lifted (see example below). In the box at the bottom, write the corresponding equation.

Original Product	Draw Half the Product (1st way)	Draw Half the Product (2nd way, **if possible**)
		
I placed 2 fingers on the bottom and 6 on the side.	I lifted *half of* my fingers from the side.	
2 x 6 = 12		
What do you notice?		
Original Product 	Draw Half the Product (1st way)	Draw Half the Product (2nd way, **if possible**)
I placed 4 fingers on the bottom and 2 on the side.		
4 x 2 = 8		
What do you notice?		

Doubling and Halving

Original Product	Draw Half the Product (1st way)	Draw Half the Product (2nd way, **if possible**)
		
I placed 5 fingers on the bottom and 4 on the side.		
5 x 4 = 20		
What do you notice?		

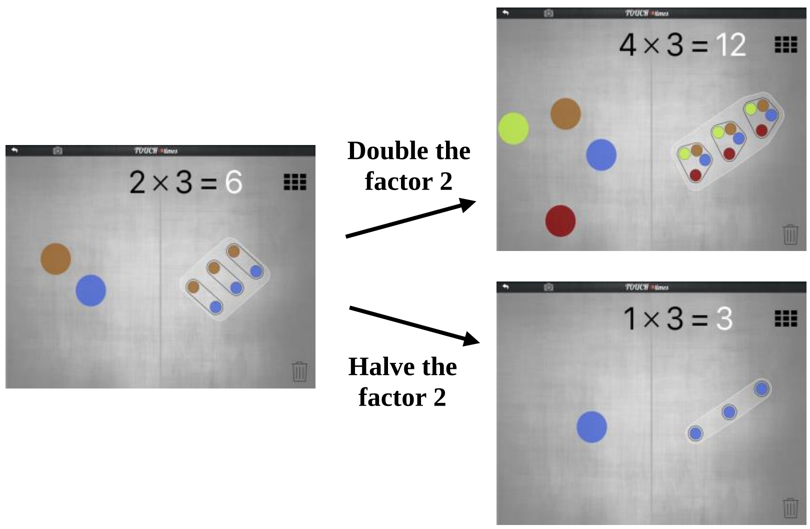
Sam makes the product 56 by using the factors 14 and 4. They want to halve 56 and thinks that one way to do this is halving both 14 and 4 at the same time. What do you think about Sam's halving strategy? Explain. If you do not agree with the strategy, what could Sam do instead?

Doubling and Halving

<u>PART B</u>

Introduction (for teacher’s reference only):

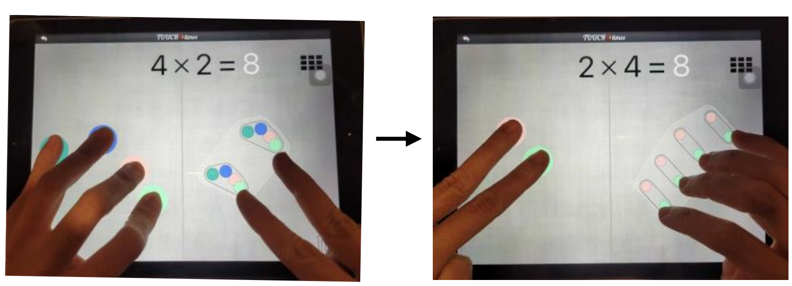
In the previous task, students explored how when you double one factor, the product is also doubled and similarly when you halve one factor, the product is also halved. This is an example of another important idea of multiplication - ***covarying*** which emphasises multiplication as a varying of two quantities. When one factor is varied and the other factor is not, the product *covaries* accordingly. For example, consider 2 x 3 = 6, when we double the factor of 2 to 4, we get 4 x 3 = 12, so the product is also doubled. Similarly, when we halve the factor of 2 to 1, we get 1 x 3 = 3, so the product is also halved.



Doubling and halving one factor at a time

Another form of ***covarying*** is when both factors can be *covaried* without varying the product. With this task, students can experience this form of *covarying* through exploring what happens to the product when we covary both factors, ie, double and halve both factors <u>at the same time</u>.

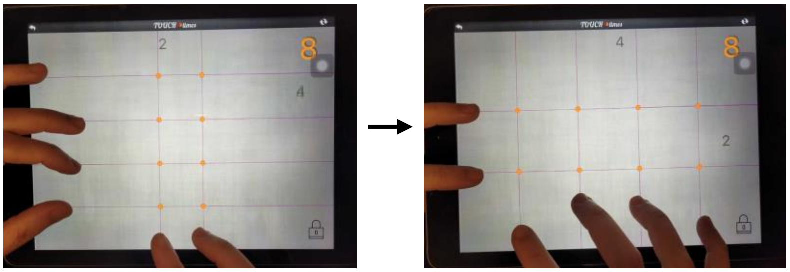
For example, consider 4 x 2 = 8, we can keep the product of 8 the same if we halve the factor of 4 and double the factor of 2. In Grasplify, we can halve the number of pips on the left by lifting two of the four fingers and double the number of pods on the right by pressing two more fingers <u>at the same time.</u>



Simultaneous halving and doubling of both factors

Doubling and Halving

Similarly, in Zaplify, when students double the number of vertical lines and halve the number of horizontal lines at the same time, they would be able to observe that the product remains the same.



Simultaneous halving and doubling of both factors

Consequently, this doubling and halving strategy can be a useful multiplication strategy to make the product easier to calculate. For example, 14 x 5 might be difficult to calculate so instead, halve the 14 and double the 5 to get 7 x 10. Why does it work? In the example, 14 can be written as 7 x 2, which means 14 x 5 can be written as 7 x 2 x 5. It does not matter what order you multiply the factors – the product is always the same, so you can multiply 2 x 5 first and then multiply the product by 7.

It is important to note that this strategy is not always helpful. For example, 9 x 7 can not be made easier by halving and doubling as one number would become a decimal. Typically, the strategy works best when at least one factor is even and the other number is a multiple of 5.

Doubling and Halving

Part 3 - In this part of the task, you will work with a partner to solve different multiplication questions. Use the doubling and halving strategy to make them easier.

Create each product in TouchTimes with your partner and then change the factors without changing the product, so that you are doubling one factor and halving the other. **Be careful!** To keep the product unchanged, you and your partner have to change the factors *at the same time*.

For each product, use arrows to show how you kept it unchanged (down arrow for halving and up for doubling - see below). For example, if you are given 4 x 2 = 8, you can keep the product unchanged if one partner doubles one of the factors, say 4, and the other partner halves the other factor, which is 2.

$$\overset{8}{\overset{\uparrow}{\cancel{4}}} \times \underset{1}{\underset{\downarrow}{\cancel{2}}} = 8$$

Here are the products. Create them on TouchTimes and don't forget to use the arrows to show how you changed the factors.

4 x 6 = 24	5 x 4 = 20	6 x 1 = 6	3 x 8 = 24

Here are some more products. Do these without using TouchTimes but continue using the arrows to show how you changed the factors.

3 x 16 =	14 x 4 =	5 x 12 =	14 x 5 =