Introduction (for teacher's reference only):

Imagine that your students were working on the following problem:

A bunch of buttons fell on the floor. Nick gathered them in heaps of 5 buttons. He made 8 heaps. How many buttons are there?

You overheard one student saying:

The answer must be odd because we are multiplying by an odd number! There is an odd number of buttons in each heap.

This is not uncommon, where students only look at one of the factors to determine if the product is odd or even. How can we help this student and possibly many other students to understand that both factors are important in multiplication and both factors affect what the product is and its parity (odd or even).

One important idea of multiplication is *covarying* which emphasises multiplication as a varying of two quantities. When one factor is varied and the other factor is not, the product *covaries* accordingly. For instance, when you multiply two odd factors, you get an odd product, but when you change one factor to be even, the product changes and becomes even. For example, consider $3 \times 5 = 15$, an odd product. When we change the factor 5 to 6, going from odd to even, we get $3 \times 6 = 18$, an even product. Thus, for this task, students will explore how they can vary the factors to obtain odd and even products and to discover the relationships between products of odd and even numbers.

Some students may be aware of these relationships, but they may not be able to explain why those relationships are true. In Zaplify, by considering the points of intersection as representing numbers, students can use the inherent arrangement or structure of the points to explain if the products are odd or even.

For example, for $3 \times 3 = 9$, when we try to divide the 9 points of intersection into two equal groups, there is a remainder of 1. Thus, 9 must be an odd number.

Or when we try to pair up the 9 points, one of the points of intersection is not paired up, so 9 must be an odd number.





Students can take a screenshot and annotate on it to explain their ideas. To take a screen capture, go to settings, click on "Image Capture Button".

Product of Odd and Even Numbers

Similarly, in Grasplify, students can easily rearrange or restructure the pods to check and explain if a product is odd or even. For $3 \times 10 =$ 30, we can divide the 30 pips on the right into 2 groups with equal number of pips in each group.



With their inherent structures, both Grasplify and Zaplify are helpful and efficient for students to explore the parity (odd or even) of numbers.

Task: Multiplying Odd and Even!

Part 1 - With the help of Zaplify, form 5 examples of an **odd** product. Record the two factors and the product in the table below:

	Factor 1	Factor 2	(Odd) Product
1			
2			
3			
4			
5			

We can use Zaplify to help us explain why the products are odd. For example, for $3 \times 3 = 9$, when we try to divide the 9 points of intersection into two equal groups, there is a remainder of 1. Thus, 9 must be an odd number. Or when we try to pair up the 9 points, one of the points of intersection is not paired up, so 9 must be an odd number.





Draw one of your odd products and explain why it is odd.

What do you notice about the factors when you have an odd product?

Product of Odd and Even Numbers

Part 2 - With the help of Zaplify, form 5 examples of an **even** product. Record the two factors and the product in the table below:

	Factor 1	Factor 2	(Even) Product
1			
2			
3			
4			
5			

Similarly, draw one of your even products and explain why it is even.

What do you notice about the factors when you have an even product?