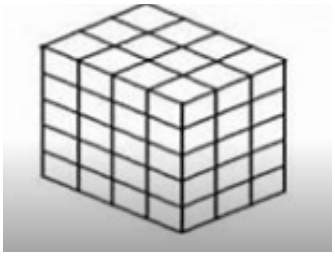
Strategies for Multiplication (Commutative & Associative Property)

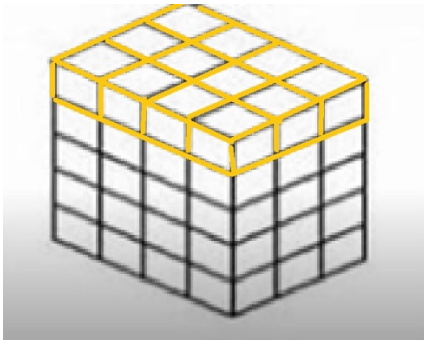
Goal of the Task (for teachers' references only)

Imagine you brought the following prism to your class and ask your students to find the number of cubes in it which can be expressed as 3 x 4 x 5. Here are a few possible answers:

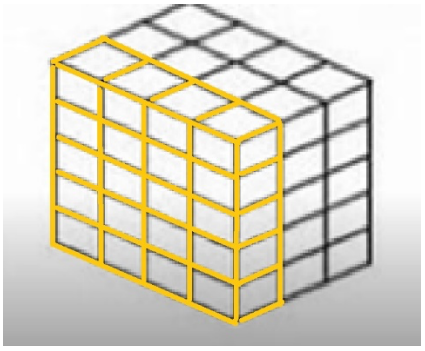


Amy: Oh, I know each layer (she refers to the top layer as in the below figure) has 3 rows and each row has 4 cubes. So, 3 times 4 gives me the number of cubes in one layer, which is 12. Since there are 5 layers, I can multiply 12 by 5. Wait I need to use pencil and paper at this point (long pause). It makes 60.

Tedy: I can find it much faster! Listen, each layer (she refers to the front layer as in the below figure) has 5 rows, and each row has 4 cubes. So, 5 times 4 gives me the number of cubes in one layer, which is 20. Since there are 3 layers, I can multiply 20 by 3 on my mind. I know 2 times 3 is 6. So, 20 times 3 must be 10 times 6 which is 60.



Amy's perspective



Tedy's perspective

Tedy finds the correct answer by using a flexible strategy. However, it is not easy for all students to discover these strategies. The primary aim of this task is to help students develop flexible strategies for multiplication. In order to use these strategies, students should be familiar with the commutative and associative properties of multiplication. These properties basically mean that the order and grouping of the factors does not change the product.

Multiplication is commutative because changing the order of factors does not change the product. For example, multiplying 3 by 5 or the vice versa will give us the product of 15. This can be shown as

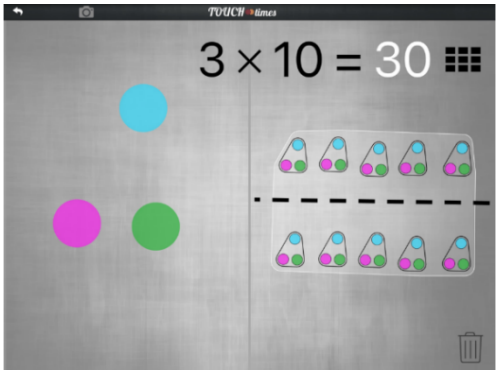
$$3 \times 5 = 5 \times 3$$

Multiplication is also associative because the product of three or more numbers remains the same irrespective of the way they are grouped. For example, given $3 \times 5 \times 2$, we can find the product by first multiplying 3 and 5 and then multiplying their product with 2, that is $(3 \times 5) \times 2$. Also, we can multiply 5 and 2 first and then multiply their product with 3, that is $3 \times (5 \times 2)$. In both cases, we will get the same product which can be shown as

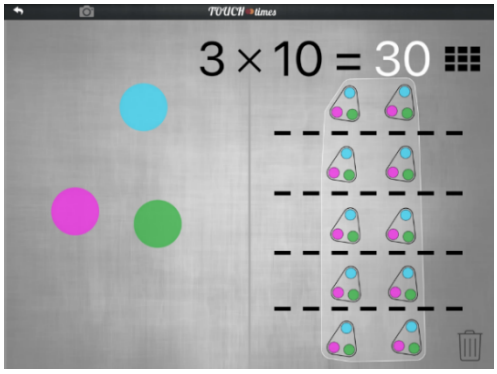
$$(3 \times 5) \times 2 = 3 \times (5 \times 2)$$

These properties help students multiply numbers faster. Instead of multiplying a list of numbers in the order in which they're written, students can group them differently to make products easier to multiply with, like 10 in the above example.

In Grasplify, we can think about and visualize the commutative and associative property by considering the different arrangements or structures of the pods as in the below figures.



(3 x 5) x 2



(3 x 2) x 5

We drew the dotted lines on the screenshots of TT to visualize the sections.

For example, the pods in 3 x 5 x 2 can be arranged by placing the pods into two sections each having five pods. This can be written as
(3 x 5) x 2. Alternatively, one can place the pods into five sections each having two pods. This is written as (3 x 2) x 5.

Again, we want students to recognize that the product does not change no matter how you rearrange the pods and hence the order and grouping of the multiplication is arbitrary.

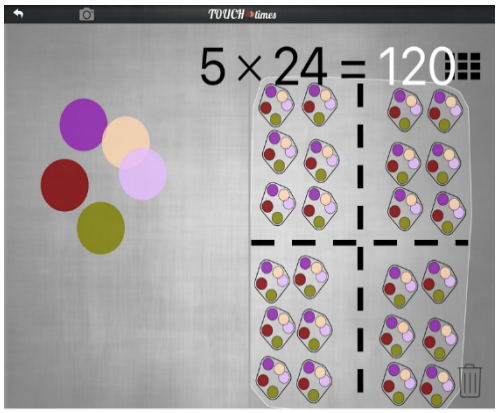
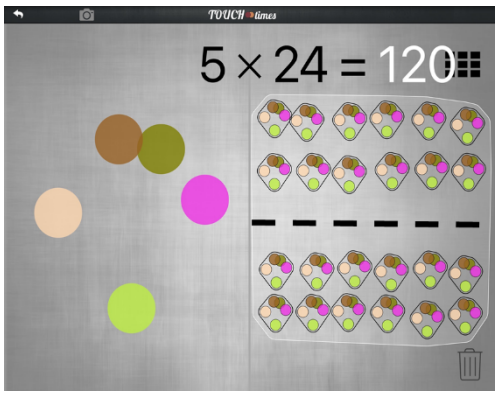
Task: Rearranging factors

Two students were discussing their strategies to perform the multiplication of 5 x 24:

Jude: 5 x 24 is the same as 5 x 6 x 4 which is then 30 x 4.

Asa: Wait! 5 x 24 is also 5 x 12 x 2 which is then 60 x 2.

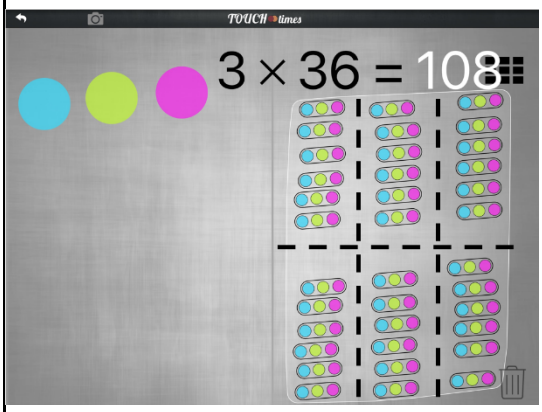
The students used Grasplify and arranged the pods to explain their strategies. Jude placed the 24 pods into 4 sections, with each section having 6 pods while Asa separated the 24 pods into 2 sets of 12 pods each.

	Strategy	Image
1	**5 x 6 x 4 = 30 x 4**	
2	**5 x 12 x 2 = 60 x 2**	

Using Grasplify, explore the different ways you can structure and arrange your pods to help you compute 5 x 24. Draw how you have arranged the 24 pods and write the multiplication expression that represents your arrangement.

	Image	Multiplication Expression
1		
2		
3		

Using Grasplify, create the product **3 x 36** and arrange them in different ways to help you compute it more easily. See the first example and then try to do the rest yourself.

	Image	Multiplication Expression
1		3 x 6 x 6 = 18 x 6
2		
3		
4		

How can you rework the following multiplication to make it easier to compute?

1. **5 x 18 =**

2. **3 x 21 =**

3. **27 x 2 =**

4. **25 x 16** =

5. **36 x 75** =