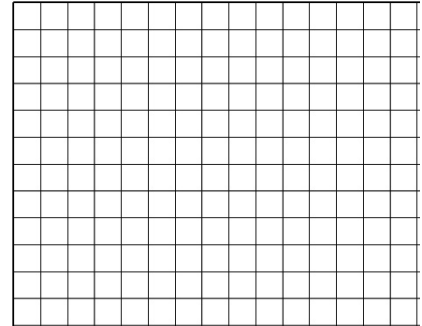


## Strategies for Multiplication (Distributive Property)

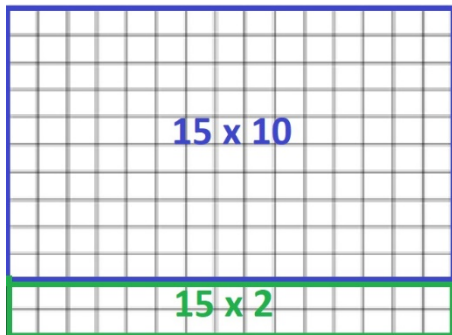
### Goal of the Task (for teachers' references only)

Imagine that you printed some bingo scorecards for your students to play 90-ball bingo. However, you need to prepare some papers to cover the numbers that would be called out. So, you also brought a graph paper as in the image on the right and asked your students how many numbers can be covered if you cut out all the small squares out of that graph paper. Here are some possible answers:



Kelly: There are 12 rows, and each row has 15 squares. So, we need to multiply 12 by 15. This is too difficult so I will use my calculator.

Nelly: I can find it without a calculator. Instead of multiplying 12 by 15, I would multiply 15 by 10 which is 150. Then I would multiply 15 by 2 which is 30. Finally, I can add 150 and 30 together, which is 180.



Nelly finds the correct answer by using a flexible strategy which can be visualized as the image on the left. However, it is not easy for all students to discover these strategies. The primary aim of this task is to help students develop flexible strategies for multiplication that are based on the distributive property of multiplication.

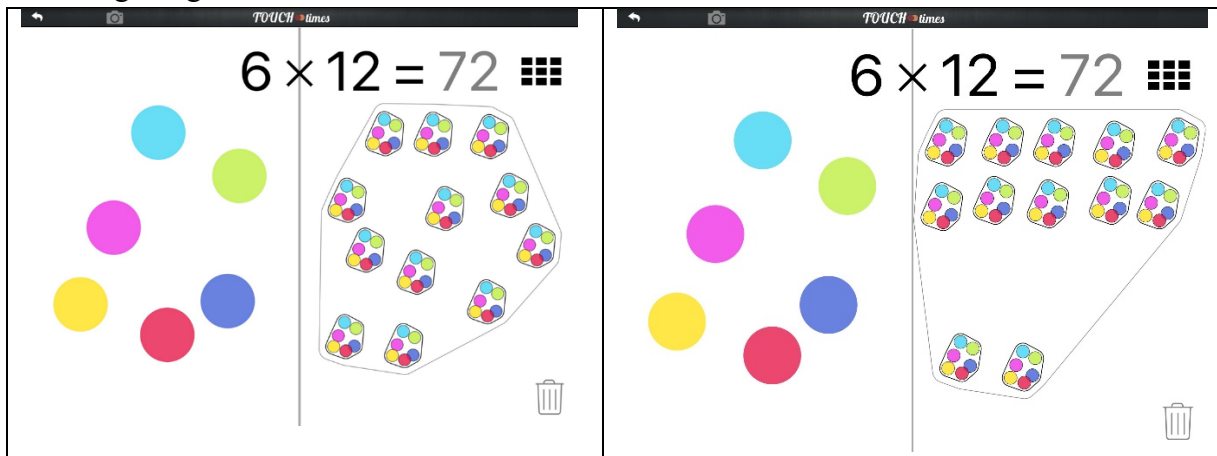
Multiplication is distributive on addition because when a number, say 2, is multiplied by the sum of two other numbers, say 5 and 4, the product is the same as multiplying 2 with each addend (5 and 4) and then adding the sum. This can be shown as

$$2 \times (5 + 4) = (2 \times 5) + (2 \times 4)$$

In this case we say, the multiplication of 2 distributes to 5 and 4. The distributive property is helpful to solve tough multiplication problems where one of the factors is huge. For example, if students want to find the product of  $15 \times 12$ , they can split 12 into 10 and 2 and calculate the simpler products as expressed in the columns in the following table.

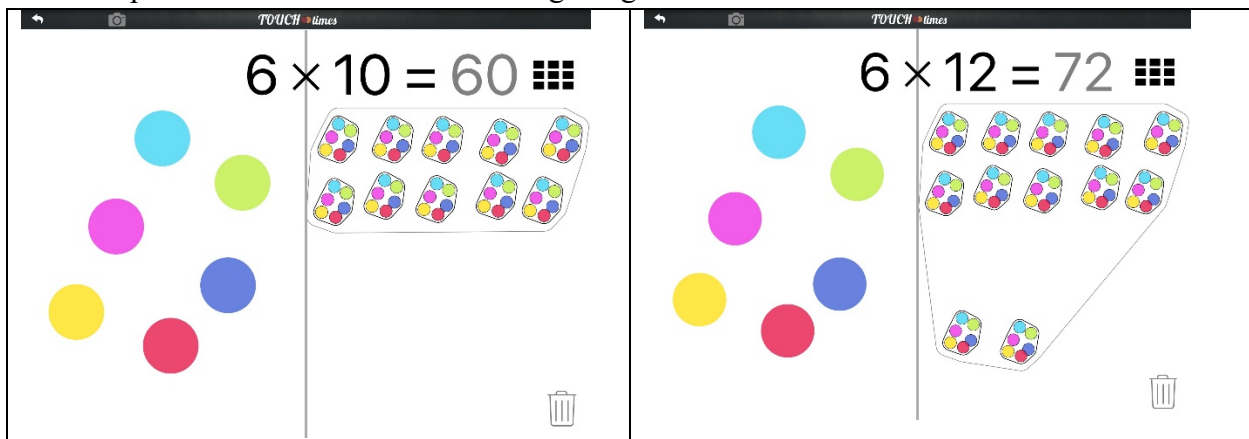
x	15
10	
2	

In Grasplify, students explore this property by rearranging or creating pods. For example, once a product  $6 \times 12$  is made, the pods can be split into two groups with sizes 10 and 2 as in the following image:



This relates to an important multiplication idea called *disembedding* the product, which is a special kind of decomposition used in multiplication. If we designate the product as the larger unit (with size 12), rearranging the pods means separating subunits (with sizes 10 and 2) from the larger unit.

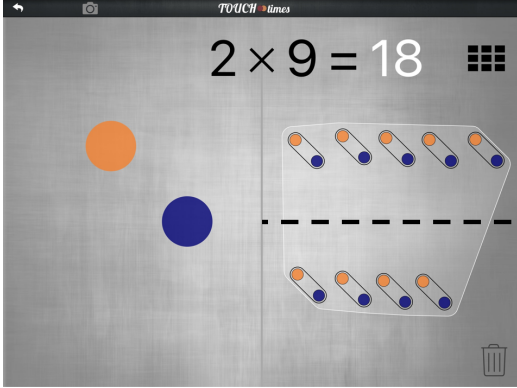
On the other hand, students may also use this property by creating new pods to increase the product. For example, once a product  $6 \times 10$  is made, they can create a group of two pods to make the product  $6 \times 12$  as in the following image:



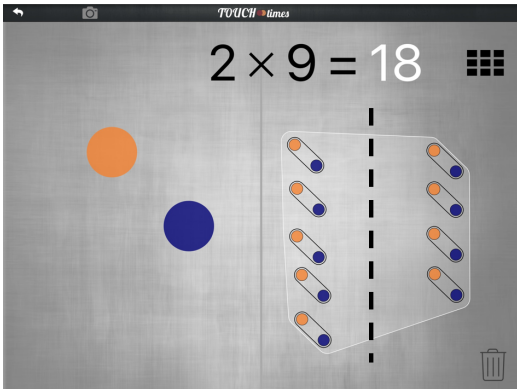
In terms of disembedding, combining pods means joining the subunits (with sizes 10 and 2) to form a larger unit (with size 12). We want students to recognize that the product does not change no matter how you rearrange the pods.

### Part 1: Divide and conquer!

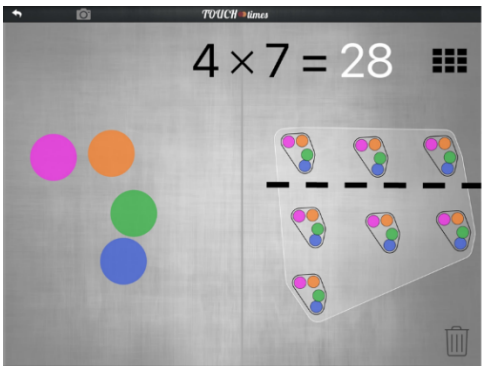
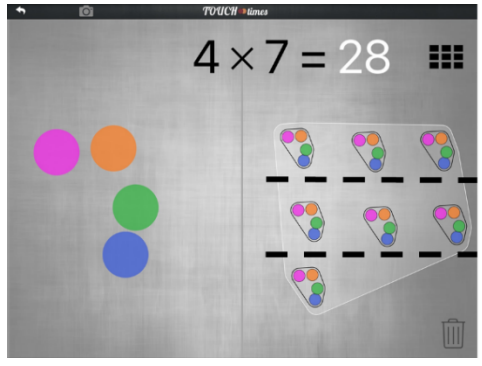
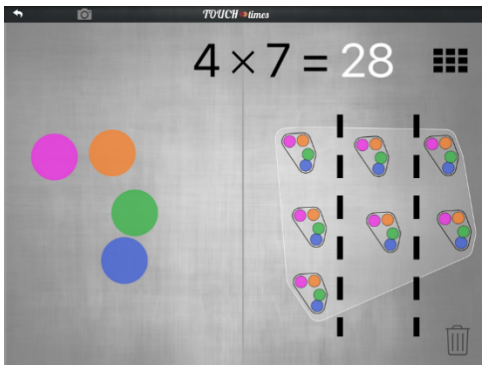
If we are given one multiplication expression, we can separate the expression into two or more multiplication expressions. For example, if we added dotted lines on the product ( $2 \times 9$ ) to separate the pods into two sections, we can rewrite the product to represent this separation with two multiplication expressions as in the following example.

	$2 \times 9$ $= (2 \times 5) + (2 \times 4)$
---	--

You can also separate the pods by adding vertical dotted lines as:

	$2 \times 9$ $= (2 \times 5) + (2 \times 4)$
---	--

Can you do the same to rewrite  $4 \times 7$  according to the given separations of 7 pods:

	Image	Multiplication Expressions
1		$4 \times 7$ =
2		$4 \times 7$ =
3		$4 \times 7$ =

Are there more ways to rewrite the product  $4 \times 7$ ? Can you draw some other ways to separate the 7 pods?

How can you rewrite  $33 \times 28$  as the sum of two or more multiplicative expressions?

1.  $33 \times 28 =$  \_\_\_\_\_  $+$  \_\_\_\_\_

2.  $33 \times 28 =$  \_\_\_\_\_  $+$  \_\_\_\_\_

3.  $33 \times 28 =$  \_\_\_\_\_  $+$  \_\_\_\_\_  $+$  \_\_\_\_\_

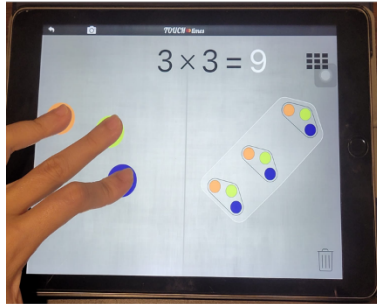
4.  $33 \times 28 =$  \_\_\_\_\_  $+$  \_\_\_\_\_  $+$  \_\_\_\_\_

5.  $33 \times 28 =$  \_\_\_\_\_  $+$  \_\_\_\_\_  $+$  \_\_\_\_\_  $+$  \_\_\_\_\_

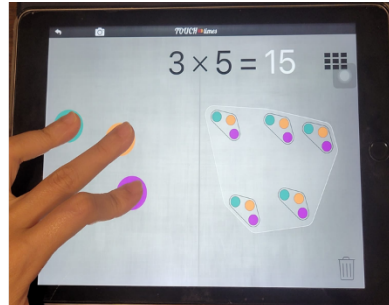
If you want to determine the product of  $33 \times 28$ , which way of separating the expression would you use and why? Use your favorite way to determine the product.

## Part 2 – Combine and conquer!

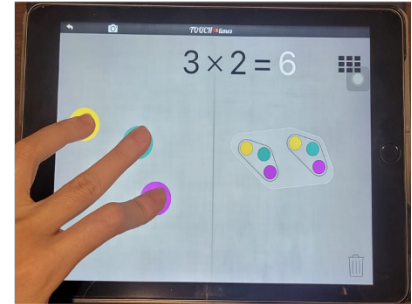
Now instead of separating pods you will combine them by using three iPads. Imagine that Hunter, Rowan and Skyler have each created a multiplication expression on Grasplify:



Hunter

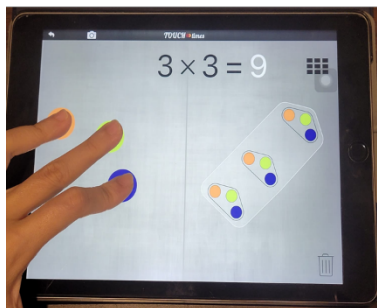


Rowan

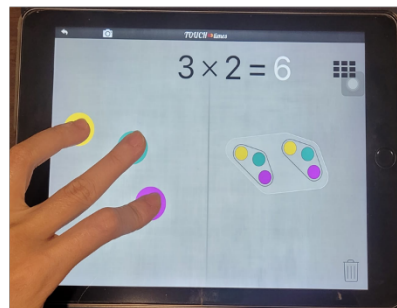


Skyler

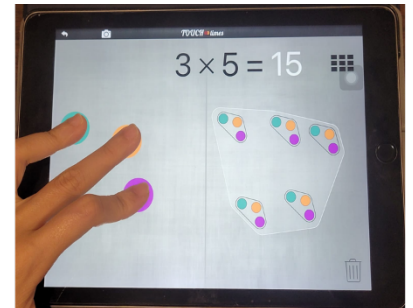
If Hunter and Skyler combine their multiplication expressions, they can together create Rowan's multiplication expression. That is:



+



=



$$(3 \times 3)$$

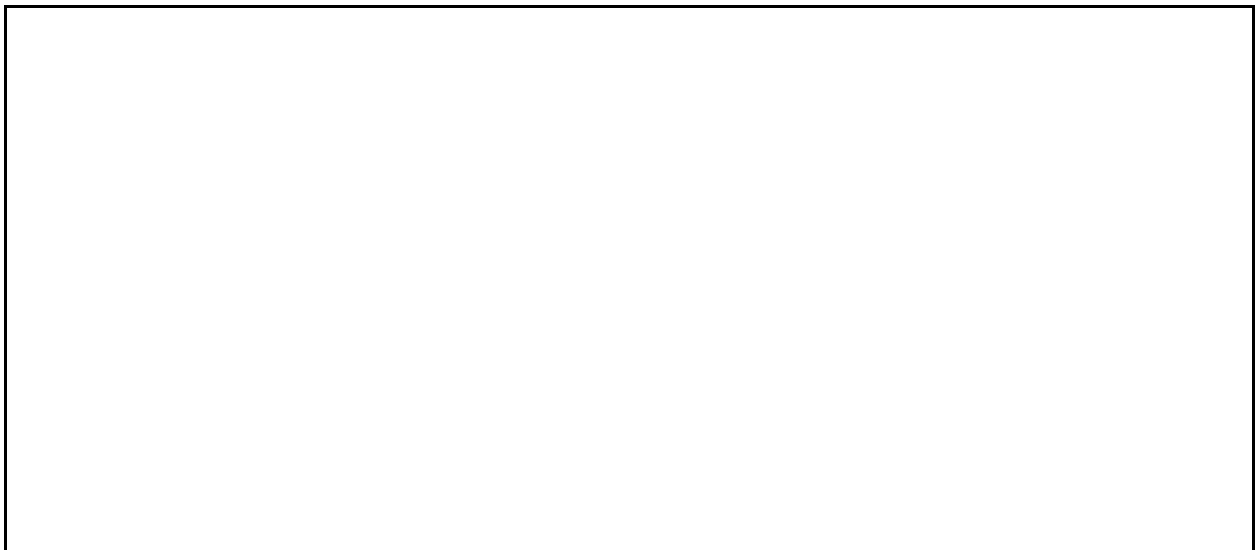
+

$$(3 \times 2)$$

=

$$3 \times 5$$

What happens if three of them combine their multiplication expressions. Draw what you see on your iPad after you have combined them.



In your groups of three, using Grasplify, create three different multiplication expressions individually first. Draw them in the boxes below. Then combine them into *one* multiplication expression and draw it in the boxes.

1	4 x _____	4 x _____	4 x _____
<b>Combined:</b>			
2	5 x _____	5 x _____	5 x _____
<b>Combined:</b>			

What did you do on Grasplify to combine the multiplication expressions?

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How could you express the following as one product (only one multiplication sign)?

1.  $8 \times 5 + 8 \times 4 =$

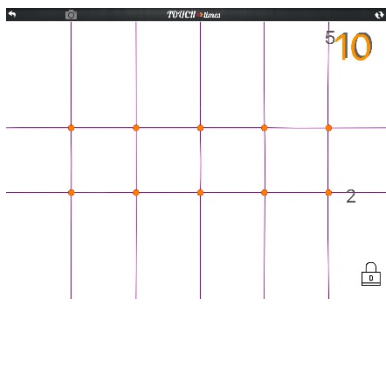
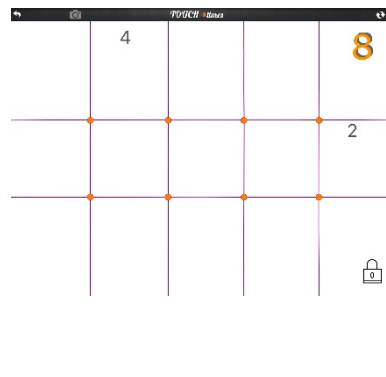
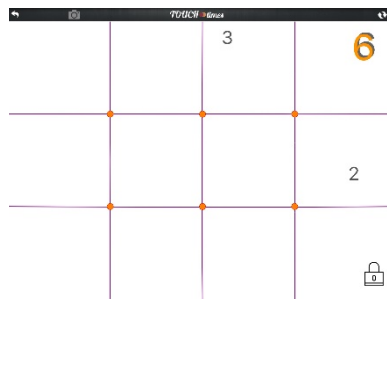
2.  $54 \times 10 + 22 \times 54 =$

3.  $132 \times 7 + 21 \times 132 + 132 \times 49 =$

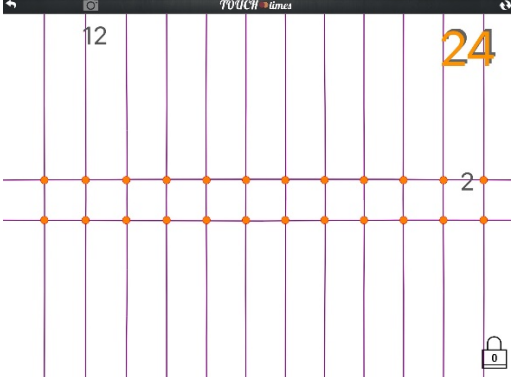


### Part 3: Match Madness!

In this task you will be given a card with a product written on it. Using Zaplify, create the product given to you. Use the lock function to keep the product on your screen! Go find other products and combine them to form another person's product! For example,  $2 \times 5$  can be combined with  $2 \times 4$  and  $2 \times 3$  to form  $2 \times 12$  as follows.

$2 \times 5$	$2 \times 4$	$2 \times 3$
 A Zaplify grid with 5 vertical lines and 2 horizontal lines. The number 10 is in the top right, and 5 is above it. A lock icon is in the bottom right.	 A Zaplify grid with 4 vertical lines and 2 horizontal lines. The number 8 is in the top right, and 4 is above it. A lock icon is in the bottom right.	 A Zaplify grid with 3 vertical lines and 2 horizontal lines. The number 6 is in the top right, and 3 is above it. A lock icon is in the bottom right.

**Combined:**

A Zaplify grid with 12 vertical lines and 2 horizontal lines. The number 24 is in the top right, and 12 is above it. A lock icon is in the bottom right.

$2 \times 12$

For teachers' reference:

$6 \times 2$	$6 \times 4$	$6 \times 8$	$6 \times 12$	$6 \times 26$
$3 \times 5$	$7 \times 5$	$1 \times 5$	$15 \times 5$	$4 \times 5$
$9 \times 9$	$9 \times 5$	$9 \times 19$	$9 \times 1$	$9 \times 4$
$4 \times 8$	$4 \times 20$	$4 \times 4$	$4 \times 5$	$4 \times 3$
$5 \times 8$	$17 \times 8$	$6 \times 8$	$2 \times 8$	$4 \times 8$
$3 \times 15$	$3 \times 2$	$3 \times 7$	$3 \times 5$	$3 \times 1$