TOUCHtimes
TASK BOOKLET

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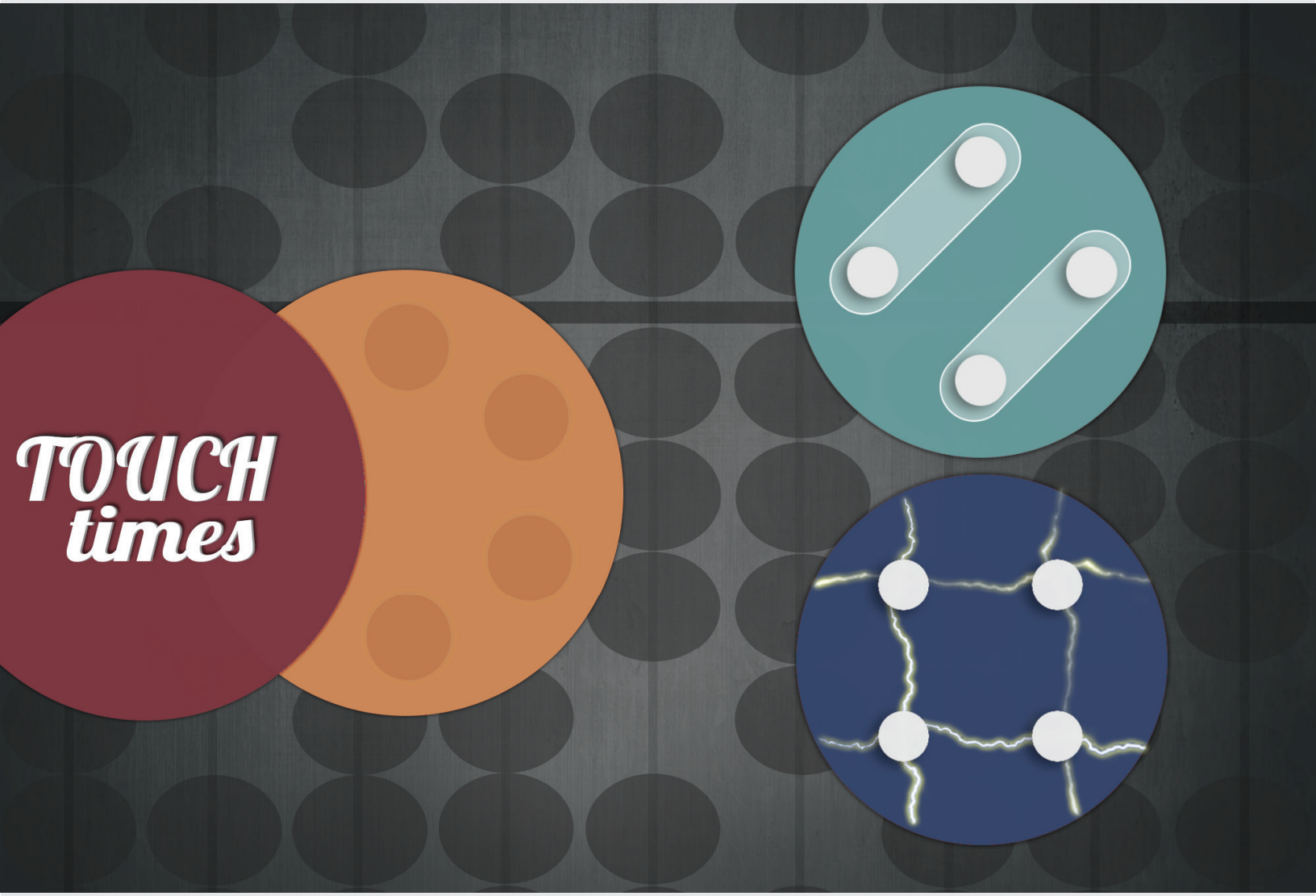


British Columbia Edition

Acknowledgements

TouchTimes was developed by the Tangible Mathematics Project under the research direction and project co-ordination of Dr. Nathalie Sinclair and the application design and software development of Nicholas Jackiw. The development of *TouchTimes* was made possible by an Insight Grant from the Social Sciences & Humanities Research Council awarded to Dr. Nathalie Sinclair (PI) and Dr. Sean Chorney (co-PI). Research assistants on the project are Sandy Bakos and Canan Güneş. Thanks also to Sheena Tan for revisions to the task booklet and Nadège Jackiw for implementation.

See http://touchcounts.ca/touchtimes/index.html for more information.

We would like to thank the many teachers who assisted in shaping the tasks found in this book. These include: Rebecca Cohen, Lisa Stringer, Alana Underwood, Victoria Guyevskey, Rebekaah Stenner, Adam Fox, Sarah Wong, Lisa Pedersen, Leah Marble, Alex Sabell, Chelsea Hinkson, Jessica Marshall, İlknur Yılmaz and Ülkü Buttanrı.

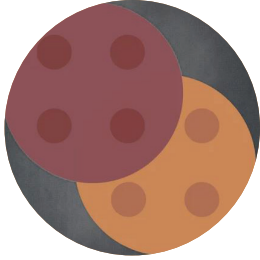


Table of Contents

TOUCH times

Introduction

Are you looking for new ways to teach multiplication? Are you interested in moving away from procedural approaches that emphasise memorisation and towards conceptual approaches that develop understanding? Are you aware that many students rely heavily on multiplication strategies based in additive thinking and how this might create problems for their future learning?

Multiplication is an important mathematical concept that many children have difficulty learning. Though there are various strategies that students can use when learning their times tables, through the use of *TouchTimes,* we want children to develop an understanding of the underlying multiplicative relationships that are an essential part of multiplication. We do this in part by allowing learners to focus on working with visual and dynamic structures that highlight the relations between factors and their product. *TouchTimes* also uses a novel, tangible approach that engages children through both visual interactions and also kinaesthetic ones. Recent research has shown kinaesthetic interactions with touchscreen technology to be very significant for children's learning. With this in mind, through actions and imagery, *TouchTimes* was developed to help students think multiplicatively, while developing flexibility and fluency with multiplication. Rather than focus on numeric computation and procedural repetition, this learning tool aims to endow children's mathematical ideas with the representational power of their fingers.

What is Multiplication?

Extending beyond counting and additive reasoning, multiplication involves a variety of skills and abilities that centre around relationships of quantity. In order to clarify what multiplicative thinking involves, we outline four key ideas that underly multiplication: **unitising**, **multi-plying**, **covarying** and **spreading**. These ideas overlap in a variety of ways.

Unitising: The idea of **unitising** is an essential multiplicative skill that involves the ability to consider a set of countable items as a single countable item, such as seeing a set of four wheels on a car as a single unit. Multiplication can be thought of as **double unitising**. You first form one unit (the number of wheels on a car) and then **unitise** the number of cars in order to find the total number of wheels.

Multi-plying: Multiplication is an action that simultaneously creates multiple versions of an original unit. Similar to a carbon copy, the original is reproduced or replicated multiple times all-at-once. This contrasts with the action of sequentially adding copies, one after another.

Covarying: As opposed to addition, the quantities in multiplication have a specific predictable relationship with each other. When one factor is varied, the product **covaries** with respect to the other factor. This emphasises multiplication as a varying of two quantities.

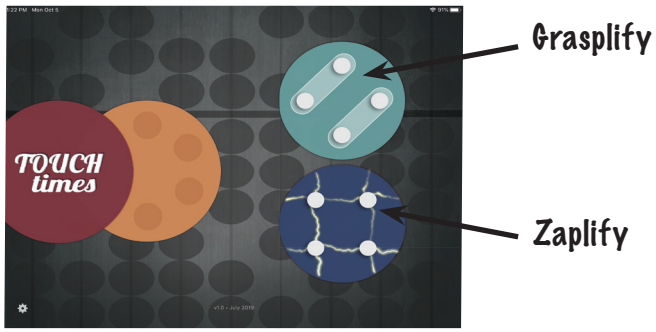
Spreading: This idea emphasises scaling or enlarging. **Spreading** can be seen when a single unit is varied and that variation is seen to **spread** across all other units.

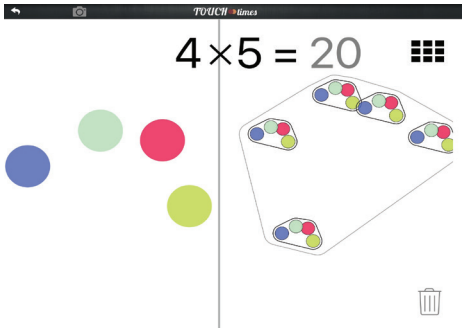
Welcome to TouchTimes

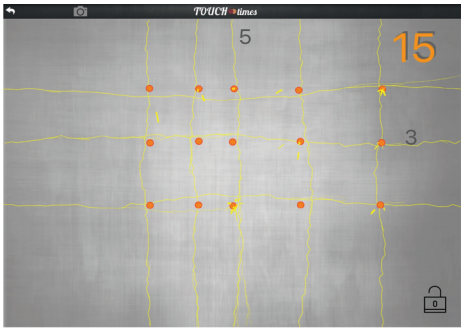
In *TouchTimes*, children use their fingers to multiply. By using two-handed gestures to create their own factors and products, learners develop a strong sense of multiplicative relationships. Since *TouchTimes* displays the product that results from any multiplicative expression, learners focus more on how, using their hands, to co-ordinate and combine multiplicative factors into given multiplicative products. Learners also encounter and work with numbers, symbols and equations, while moving back and forth between symbolic/ numeric forms and more visual, haptic and tactile representations of multiplicative relationships.

TouchTimes offers two complementary experiences in the Grasplify world and the Zaplify world, each embodying a different multiplicative model. Grasplify is based on a pip-pod (multiplicand-multiplier) model, whereas Zaplify is expressed by orthogonal intersecting lines, similar to an array model. Both worlds involve a two-handed gestural approach for exploring and symbolically expressing **unitising**, **multi-plying**, **covarying** and **spreading**.

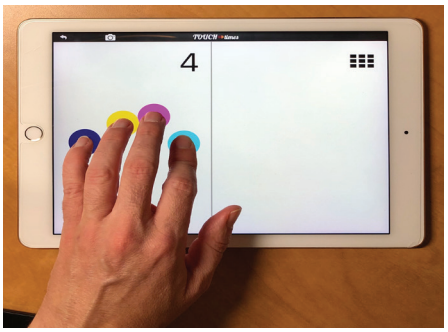
A more detailed description of each microworld is in the pages that follow, but experiences with each world allow learners to compare and contrast different ways of thinking about multiplication. Encountering and comparing these similarities and differences is an important part of thinking about multiplication and in order to encourage this process, we explicitly ask the same questions in both worlds. This also provides support to learners who gravitate more towards one world than the other. The experiences that learners have with the two multiplicative models in *TouchTimes* are beneficial in helping them develop more affective relations with different multiplicative models.



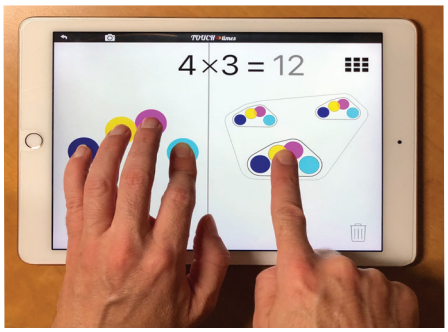




Grasplify World

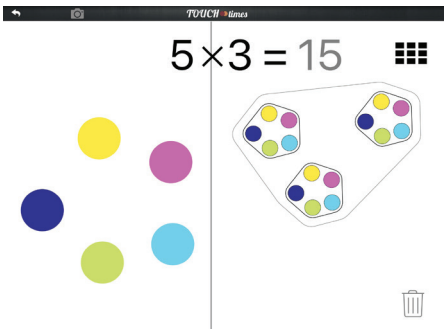


The Grasplify world enables experiences of multiplication that go beyond approaches based on additive thinking and repeated addition. In this world, learners experience the **double unitisation** of multiplication. They first create with the fingers of one hand, a unit. The creation of this unit as the first action emphasises **unitising**. Then through one or more simultaneous taps of the other hand—scale that unit to a unit of units, which becomes the product, this is the action of **multi-plying** (as well as the second instance of **unitising**). Grasplify's order follows approaches to early mathematics that are grounded in measurement and ratio, where one wants to identify the unit quantity before asking "how many units?" In research circles, this is known as a Davydovian approach.



Grasplify - Where is the Math?

The Grasplify world emphasises a distinct role for each factor in multiplication, by having learners first create the things to multiply (pips, or more formally: the multiplicand), and then create the number of times by which to **multi-ply** them (pods, or the multiplier). The software displays the total number of pips in all pods as the product of these factors.

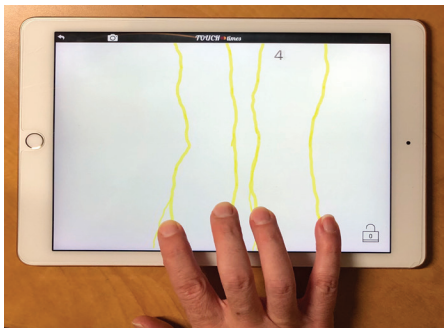


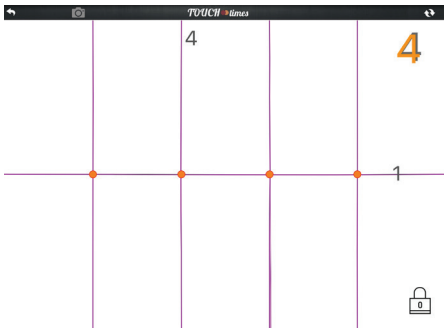
Grasplify's ordering of multiplicand-then-multiplier fits naturally with the physical nature of "pips in pods" of the software's design. In more algebraic settings, however, the multiplier frequently appears before the multiplicand, as when we read "5 x *n*" as "five groups of *n*." Each approach makes sense in its context. Thus "5 × 3" can be correctly seen as either "5 groups, each of which contains 3 things" or "5 things, in each of 3 groups." The teacher's choice of language can help support a specific interpretation of such statements in a particular context. Some teachers adopt language such as "5 pips in each of 3 pods"—or more simply "5, 3 times"—to describe Grasplify's physical model.

Grasplify embodies **unitising** and **multi-plying**. When one hand makes a set of pips, the learner is **unitising**, and when the other hand makes a set of pods all-at-once, they are **multi-plying**. To draw out the other two key multiplicative ideas, we have designed specific tasks to encourage learners to see that **covarying** the number of pips alters each and every pod, as well as the product. And similarly, **covarying** the pods alters the product. When doing the tasks, we also encourage altering the orientation and number of pips so that learners will see and experience the **spread** of these properties within each pod.

Zaplify World

In Zaplify, learners express multiplication by creating lightning bolts. The number of these vertical and horizontal lightning bolts represent multiplicative factors and the number of orange sparks that appear where they intersect, represent the product. To draw out the key multiplicative ideas in Zaplify, we have designed specific tasks that encourage learners to see the **covarying** of the factors and the effect on the product. In the Zaplify world, **unitising** is created with two hands. If four fingers are touching on the horizontal edge and one finger is touching on the vertical edge, the unit is the intersection of these lines, in this case, four points (4 x 1 or 1 x 4). With additional all-at-once touches on the horizontal or the vertical edges of the screen, intersection points are **multi-plied**.





Zaplify - Where is the Math?

The layout of Zaplify provides learners with a visual grid model—or, more generally, a geometric model—for thinking about multiplication and about how **covarying** factors affect the product. This grid model emphasises factors as being symmetric, such that the order of placing fingers does not matter. In the Zaplify world, children discover that one hand "works" much like the other!

With several fingers along the bottom of the screen producing vertical lines, the horizontal line produced by a finger placed on the left side of the screen will cut across each and every vertical line, thereby **unitising** the set of intersections. Each additional horizontal line **multi-plies** that unit by copying it along each new horizontal line.

The factors and the product are displayed numerically in distinct areas of the screen in order to focus attention on their **covarying** relationships. When a learner makes a change to one factor, the structure of the grid automatically adjusts to reflect the change. This adjustment emphasises the **spreading** of each factor across another factor, that is, when a new vertical line is created, it intersects with each and every horizontal line.

Direct learners' attention to the sparks formed by the intersections of the lightning bolts rather than the spaces created between them. Developing your own name for these—sparks, or fire-bugs, or crashes—helps invest them with identity. As in the Grasplify world, multiplicative products are already computed by the environment, so the focus here should be on patterns and relationships rather than on "what's the product?"

The Benefits of Having Two Worlds

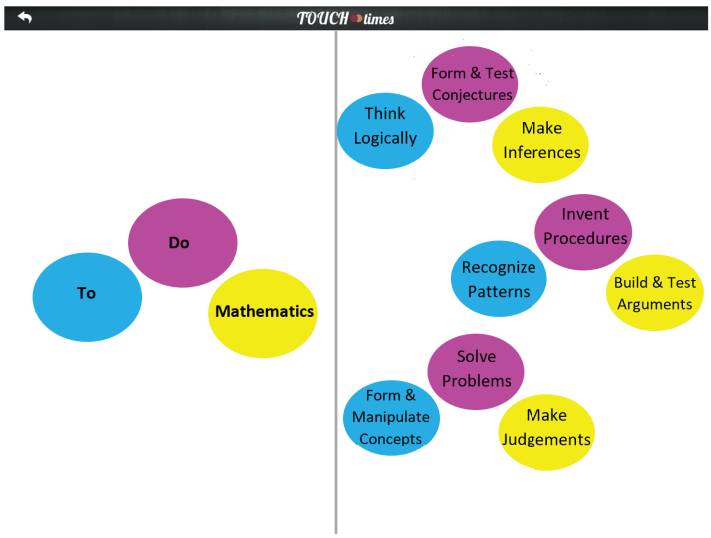
TouchTimes embodies two different multiplicative models. The Grasplify world is based on a pip-pod (multiplicand-multiplier) model and the Zaplify world is expressed by orthogonal intersecting lines, similar to an array model. Both worlds involve a two-handed gestural approach for exploring and symbolically expressing the key ideas of **unitising**, **multiplying**, **covarying** and **spreading** previously described. Both models enable learners to work simultaneously with the physical/concrete and the symbolic/formal aspects of multiplication, instead of moving from the former to the latter.

Some teachers may choose to have their students engage with the Grasplify world first, while others may begin with Zaplify. This choice may depend on the other teaching or learning resources and activities that are being used in conjunction with TouchTimes. Though the Grasplify tasks are first, followed by Zaplify tasks, teachers can choose to work through the tasks in the order given, or can choose to jump back and forth between the tasks--you may even have some students do a task in Grasplify, while others do the equivalent task in Zaplify. The Grasplify tasks are ordered to develop multiplicative thinking step-by-step, while the order of the Zaplify tasks exploits the symmetric nature of multiplication. If you choose to begin with Zaplify to build multiplication step-by-step as in the Grasplify section, we suggest the following order of the Zaplify tasks: Zaplify Exporation, Unitising, Doubling, Halving, Skip Counting, Multi-plying, Order Doesn't Matter and Maximising the Product.

Whatever choice you make, we highly recommend a final activity in which you explicitly invite students to compare the two worlds. This might involve asking them to prepare a small video about how each world works, or explaining when or why they think one world is more useful or more interesting than the other.

General Pedagogical Approach

Fundamental to the approach taken in this task booklet is the belief that free exploration and play enable valuable learning opportunities for children as they experiment with mathematical ideas. Mathematics is a form of reasoning that assists in solving meaningful problems. To do mathematics requires more than the performance of rote procedures created by others and memorised, rather it involves examining, manipulating and creating mathematical images and ideas related to quantitative or spatial problems. Using *TouchTimes*, learners create and manipulate mathematical objects with their fingertips, while making sense of what is happening on the screen. Engaging in these processes supports mathematical thinking.

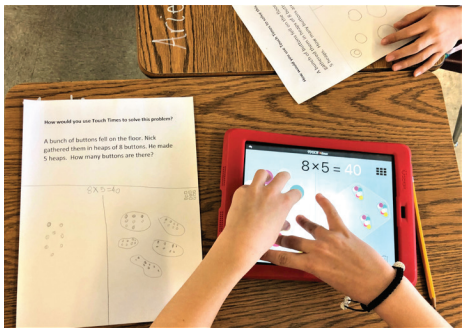


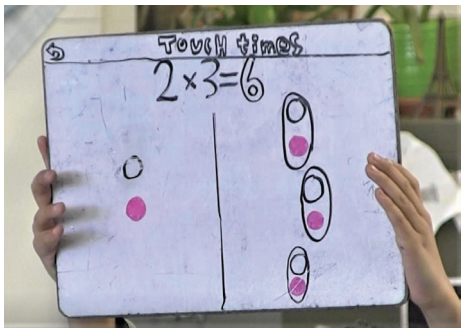
Why Work in Partners?

When children work in partners, they need to co-ordinate their actions on the screen, verbalise their strategies to each other and reflect on what they and their partner are doing. These sorts of skills are outlined in the core communication competency as forms of communication and collaboration. It is documented within that core competency that these kinds of interactions between students support acquiring, developing and transforming ideas and information, which thereby supports learning. Another advantage of working in pairs, is that it increases the number of fingers available to be used!

Getting Started

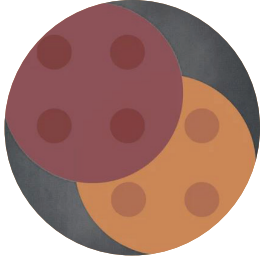
Children benefit from making choices and having a voice. *TouchTimes* encourages exploration and choice-making in an environment that is structured by mathematical rules but is more open-ended in design. The tasks in this booklet are designed to inspire curiosity and provide learners with opportunities to explore and seek solutions. The role of the teacher involves establishing a spirit of inquiry and supporting learners' experiences with *TouchTimes* by providing time for open exploration, while asking children to predict or explain *TouchTimes'* responses in different situations. Encouraging learners to explain their ideas, as well as demonstrate them, is important, as talking through one's own thinking is one of the strongest ways to develop and solidify understanding. You might try to pose small challenges for them, or to ask them questions about the mathematical challenges and goals that they set for themselves.





Once learners have had sufficient opportunity to solve and make sense of the assigned task, the teacher then brings the whole class together. This provides an opportunity for children to demonstrate, explain and justify their mathematical thinking for their peers, and allows the teacher to introduce or review mathematical concepts and terminology so that learners can use these ideas or vocabularly to communicate and to reflect on their developing ideas in subsequent lessons.

Please note that the tasks can be explored in any sequence you choose, however, be aware that many of the introductory activities are designed to review ideas developed in the previous task.



Connections to Curriculum

The activities in this booklet are designed to connect and relate with each of the Big Ideas and Curricular Competencies listed below. Teachers can narrow in on some or all of the Curricular Competencies depending on the focus for that particular class. [Note that content, content indicators and connections to Indigenizing the curriculum are embedded in each activity write-up.]

Big Ideas

Computational Fluency: (unitising, multi-plying)

- Development of computational fluency in addition, subtraction, multiplication, and division of whole numbers requires flexible decomposing and composing. [Grade 3]
- Development of computational fluency and multiplicative thinking require analysis of patterns and relations in multiplication and division. [Grade 4]

Patterning: (covarying, spreading)

- Regular increases and decreases and patterns can be identified and used to make generalisations. [Grade 3]
- Regular changes in patterns can be identified and represented using tools and tables. [Grade 4]

Curricular Competencies

Reasoning and Analysing

- Use reasoning to explore and make connections
- Develop mental math strategies and abilities to make sense of quantities
- Use technology to explore mathematics
- Model mathematics in contextualised experiences

Understanding and Solving

- Develop, demonstrate, and apply mathematical understanding through play, inquiry, and problem solving
- Visualise to explore mathematical concepts

Communicating and Representing

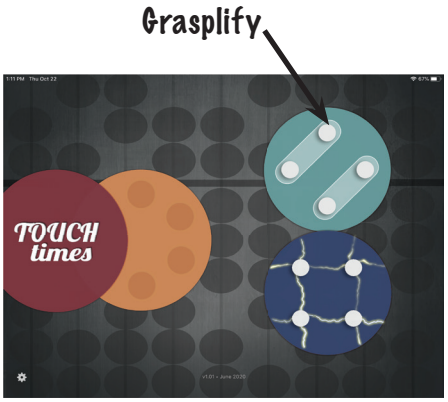
- Communicate mathematical thinking in many ways
- Represent mathematical ideas in concrete, pictorial and symbolic forms

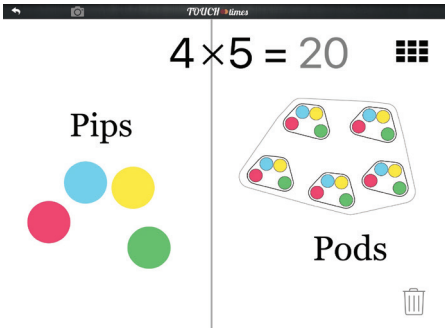
Connecting and Reflecting

- Incorporate First Peoples worldviews and perspectives to make connections to mathematical concepts (see elaboration in each activity).

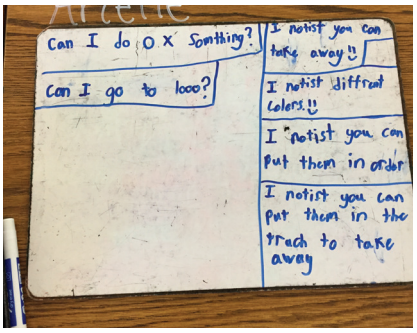
GRASPLIFY EXPLORATION

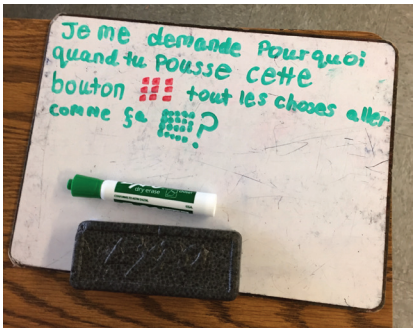
Exploration





1. Project the *TouchTimes* homepage onto a screen for the class to view. Tell students, ***Today you're going to use an app called* TouchTimes. *When you get your iPad, you can press this button (indicate which button to push) to open the Grasplify world. You will then have time to play with and explore the app. While you're exploring, pay attention to what is happening in Grasplify and jot down what you notice or things that you wonder.*** Mini whiteboards are a great resource for students to do this.





2. Allow time for students (alone or in partners) to explore the app. Assist anyone experiencing technical difficulties. Be sure students have opened the Grasplify world.

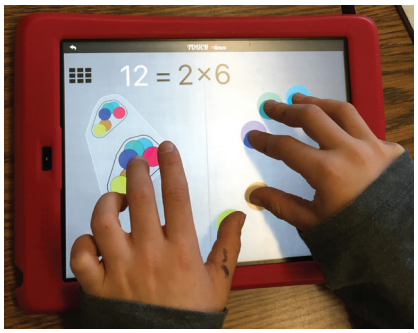
   Vocabulary Alert! Watch for opportunities to introduce and reinforce the terms pips and pods.

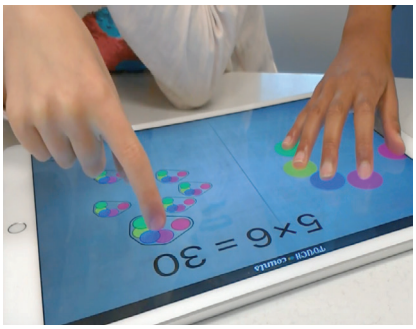
3. Ask questions to determine how students are thinking about the use of Grasplify. ***What happened when you placed your fingers down on right/left side first? What do you notice about the number sentence at the top of the screen? How did you make that happen? Can you place multiple fingers down at once (on the pip side/on the pod side)? What things did you notice about the colours?*** Encourage students to try different combinations of actions with their hands.

VOCABULARY

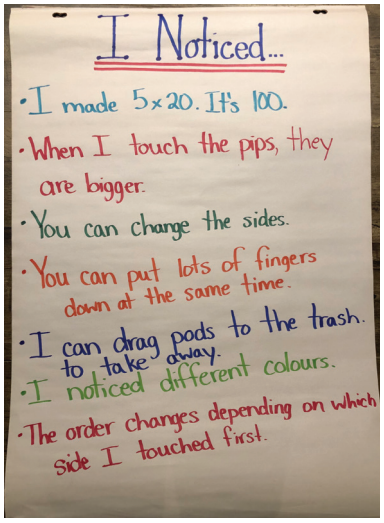
The pips represent the group size (multiplicand).

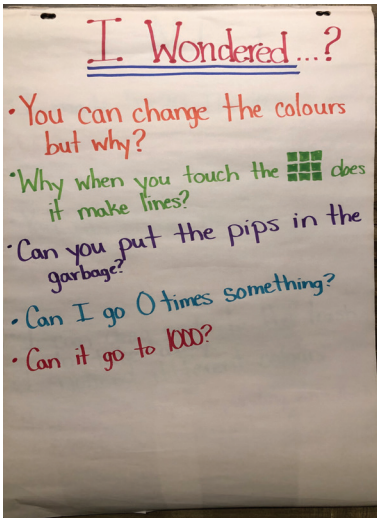
The pods represent the number of groups (multiplier).





5. Once students have had an opportunity to get a feel for how Grasplify works, gather the class together for a whole group discussion. You may want to project Grasplify on a screen, so that it is available for students to demonstrate and explain their discoveries to their classmates. ***What did you notice about Grasplify?*** As students share their ideas, record these on an I Noticed/I Wonder chart for all to see. ***Is there anything that you wonder about Grasplify?*** Record these ideas on the chart as well. Use the questions above to encourage more discussion and demonstration from the class.





TEACHER TIP

You may want to have students engage in both the Grasplify Exploration and the Pips into Pods task during the same lesson.

Pips into Pods

Video Link

Summary

This task invites students to notice the unitising relationship between the pips and each pod and the pods and the product.

Curriculum Objectives

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for this task.

Content

- Provide opportunities for concrete and pictorial representations of multiplication
- Connect multiplication to skip counting

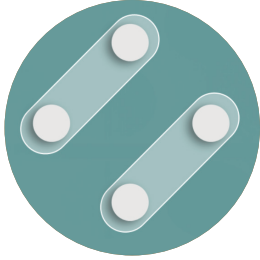
Introduction

SETTINGS

Remember to turn off the Numeric Factor Values and the Numeric Product Values in settings.

For this task, make sure that the Numeric Factor Values and the Numeric Product Values are turned off in the Settings. The Settings button can be found in the bottom lefthand corner of the main *TouchTimes* screen.

1. Project Grasplify so that all students can see the screen (if you have access to a document camera, this allows students to see what your fingers are doing on the screen as well). Place 5 fingers on the screen. Tell students that, ***These are called pips and I want to put all of the pips into pods.*** Then press 4 fingers on the other side of the screen to create 4 pods. Say, ***I'm now creating pods. I'm going to remove my fingers from the pods so that we can see them better. Notice that all of the pips are also inside each of these pods.***
2. Explain, ***Now we're going to play a game. I am going to choose two students. One student to leave the room and another student to come up here and use Grasplify to create a multiplication number sentence with pips and pods. Everyone else needs to pay close attention to everything that happens on the Grasplify screen.***
3. After which, ask [student 1] to come back in. ***Now, the class need to explain to [student 1] that left the room exactly what to do with his (or her) hands in order to recreate the multiplication number sentence that [student 1] had on the screen.***



4. Play a few rounds of this game.

5. While the students are sharing, here are some things to listen for:

    - If students have had experience with multiplication, they may describe what they see as “4 pods of 5 pips”. Encourage them to describe what they see using the pips first, so, perhaps, “5 pips taken 4 times” or “5, 4 times” or “5 pips in each of 4 pods”. If students comment on the total number of pips that are in the pods, you can bring that number—20 in this case—into the description.
    - Also ask astudents, ***How did you figure out the total?***
    - If they counted one at a time, ask students, ***Are there are other ways of counting?*** Demonstrate how it is possible to count up by fives to reach the total. Counting by fives acounts for the structure of the pips in the pods.
    - Students might also comment on the colour and shape of the pods with respect to the pips, but if they don’t, you can prompt their noticing either by asking about colour or by lifting a pip-finger and pressing it down again.

TEACHER TIP

The language used here is very important and highlights the unitising of the pips into a pod and then the unitising of the pods into the envelope.

Task

1. Ask students to model the following situation using Grasplify: ***On the weekend, I went to the Dollar store to buy some pens. When I got there, they only had packages of 5. I needed enough for our class, so I bought 6 packs. Use Grasplify to show what the packages of pens would look like. Be sure to include enough packages for our class.***

TEACHER TIP

Make sure that students have made 5 pips and 6 pods. Some will have done the inverse.

Extending Student Learning

- Provide a list of skip counting number sequences (e.g. 2, 4, 6, 8; 3, 6, 9, 12; 4, 8, 12, 16; 5, 10, 15, 20, 25) and ask students which number sequence they would use to count the pips without counting each and every pip. This way the students who are not fluent in skip counting number sequences might also find an entry point for this task.
- If students are quick to create the correct model, ask them to determine how many pens there are in total, without counting every pip.
- Additional prompts could include: ***What would it look like in Grasplify if each pack contained a different number of pens?***

TEACHER TIP

We want students to notice that by skip counting by five, we can find the total number of pens more quickly and easily than by counting each individual pen.

CAUTION!

This visual relationship requires work. Often what children see is not the same thing that we see. Watch to see that they have drawn 4 pips in 3 pods as this is the multiplicative structure in Grasplify.

2. Bring students back togther for a class discussion. Be sure to project the Grasplify world for all to see. Invite a student up to demonstrate for the class how they modelled the situation using Grasplify. Ask, ***How can we find the total number of pens (in this case, pips)?***

3. Have the students at the iPad place another pip-finger on the screen. Ask students, ***How does an extra pip, or I guess in this case that pip is supposed to be a pen. How does an extra pen change the situation in the packages?***

4. Ask students to draw the following situation, ***While I was at the Dollar store, I also saw some boxes of dry erase markers. There were 4 markers in each box, so I bought 3 boxes. Draw 4 x 3 = 12 using pips and pods the way that Grasplify would show it.***

5. Show a screenshot of Graspify that has 9 pips and 3 pods. ***Now that we've done the packages of pens and boxes of markers situations, what other kinds of situations can you come up with to fit this model?***

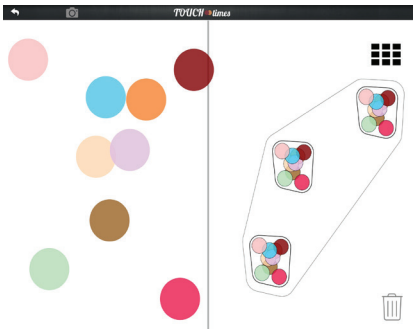
6. Say, ***Now it's your turn, you are going to create a multiplication number story and your partner is going to show what this would look like using Grasplify. Then switch. How many different stories can you come up with?***

Summarise

1. Gather students together and have a few pairs share and model their stories with the class.

Assessment

Draw the screen that shows the model of the pen-package situation and then write a number sentence that describes what is happening on the screen.



Doubling

Summary

Students create pictorial representations of multiplication using Grasplify to investigate the mathematical strategy of doubling, which is more multiplicative in nature than repeated addition.

Video Link

Curriculum Objectives

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for this task.

Content

- Provide opportunities for concrete and pictorial representations of multiplication
- Using mental math strategies such as doubling or halving
- Fluency with math strategies (e.g. identifying related doubles)
- Understanding concepts of multiplication (e.g. arrays)
- Multiplication and division are related

Indigenous Connection

Doubling, halving:

Knowledge holders in some Indigenous nations are known to utilise the concepts of doubling and halving for measuring purposes. Folding a piece of paper, string or any foldable material in half, provides a way of finding the half of a whole or is a way of determining double the length of the half. Iterating this process can double and divide into halves multiple times. Folding algorithms can be quite complex. In folding a strip of paper into 5 equal units, for example, a section at one end of the strip was estimated to be a fifth of the whole, and the remaining strip of paper was folded twice in half since this would create 4 equal units. The unfolded strip of paper, together with the original estimate would have 5 equal units.

Introduction

1. Allow time for students (alone or in partners) to explore the app. ***Today you will have a short period of time to explore Grasplify. While you're exploring, pay attention to the ideas on our What I noticed chart from last class. There may be things that your classmates noticed that you never thought of or that you didn't get a chance to try last class. Jot down anything new that you notice or things that you wonder that can be added to our chart.***

CAUTION!

Try not to rush this step. Free exploration allows for valuable learning for children as they experiment with mathematical ideas.

VOCABULARY

A product is the result of multiplication or a math name for a multiplication answer.

WHAT TO WATCH FOR

Ensure that pairs have doubled the product by changing the pips. When first given this task, many children double the product but do so by changing the pods, which emphasises repeated addition.

TEACHER TIP

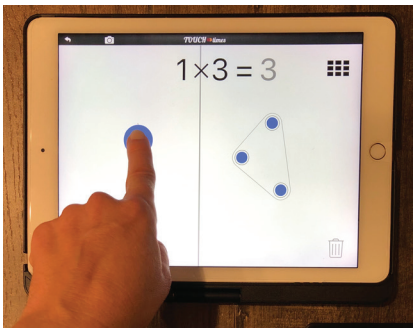
This increase in the number of pips should draw attention to the spreading effect of pips within each and every pod.

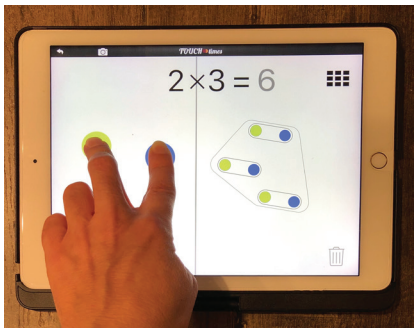
2. Once pairs have had an opportunity to explore Grasplify, gather the class together for a brief discussion. Have Grasplify available for projection if students have new noticings to demonstrate for their classmates. ***What did you notice about Grasplify that you didn't notice last class? Is there anything that you wondered that we don't already have on our chart?*** As students share their ideas, add them to the I Noticed/I Wonder charts for all to see.

Task

1. Project Grasplify so that all students can see the screen (if you have access to a document camera, this allows students to see what your fingers are doing on the screen as well). Emphasise making the product $1 \times 3 = 3$ using one pip and three pods. If you don't have access to a document camera, you can take a screenshot of $1 \times 3 = 3$ in order to demonstrate with your fingers on the projected screen image how to create one pip and three pods. ***Your challenge today is to double the product so that it is six. What is a product again? Right now, the product is three. How can you double the product so that it is six without changing the number of pods? See if you can figure out how to make the product six by changing ONLY the number of pips. Once you've done that, double the product again to make 12. Then double it again to make 24.*** Write the sequence 3, 6, 12, 24 on the board for students to refer to while working.

   One way of doubling is to place an additional pip-finger on the screen. In the example below, the product doubles, going from 3 to 6.





2. As you circulate, observe and listen to students' conversations. Here are some things to watch for:

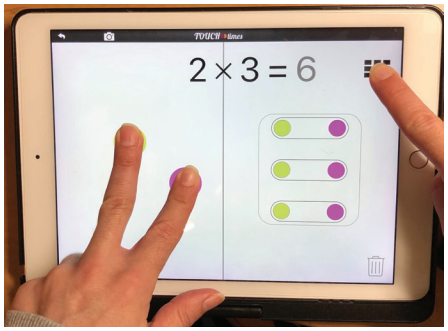
   - Students will have to shift from increasing the number of pips by 1, to doubling the number of pip-creating fingers from two in $2 \times 3 = 6$ to four in $4 \times 3 = 12$ and then to eight in $8 \times 3 = 24$.
   - Find other ways of getting from a product of 3 to a product of 24. Leave this open ended to see what students come up with.

Extending Student Learning

- Early finishers can be challenged with, ***Can you continue to double the product by changing the number of pips?***
- Invite students to triple the product. They will quickly run out of fingers, so it will be necessary to imagine how to triple the product and they can document their thinking on paper.
- Early finishers can also explore some of the I Wonders from the anchor chart.

Summarise

1. Once the pairs have successfully doubled 1 × 3 = 3 by changing only the pips, to create products of 6, 12 and 24, gather them together for a class discussion. Be sure to project the Grasplify world for all to see.
2. Have a pair create 1 × 3 = 3 on the iPad and then demonstrate how to double the product to make 6. Encourage students to verbally state each time they place a pip-finger on the screen. Ask students, ***What happened when [insert student's name] increased the number of pips?*** Ensure that children explain what happened within the pods when another pip was created. ***What colour is the pip that [insert student's name] just made? Where else do you see pips that colour? Do you notice anything about the shape of the pips?*** Draw attention to the colour and shape configuration of pips and how this is reflected in the pods.
3. Say, ***We now know how to double a product of three to make six.*** Point to the array button and ask, ***Did anyone notice this button on their screen? When you push this button, it will create an array. Notice how Grasplify organised 2 × 3 = 6 into an array with three rows and two columns. There are three in one column and three more in the second column, which shows three doubled. What is 6 doubled? How would we double six to make a product of twelve?*** Have a new pair come up and create 1 × 3 = 3. Have them double the product to make 6, then demonstrate and explain how to double 6 to make a product of 12. Again, draw students' attention to the array button and how it now shows 4 × 3 = 12 in 4 columns and 3 rows. Repeat the process by doubling 4 pips to make 8 pips and the final product of 24. Draw the students' attention to both the colour and configuration of the pips and how the added pips **spread** (see photos on the next page) across the pods.

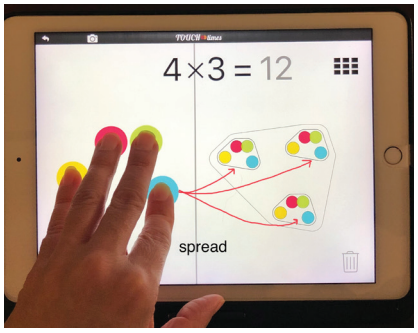


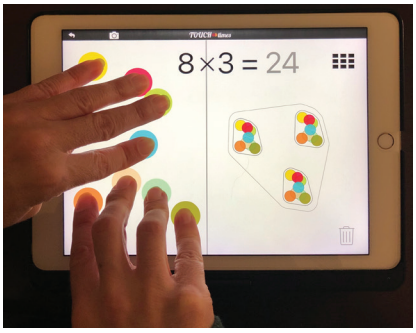
TEACHER TIP

This is a great opportunity to introduce students to the array button and explain what arrays are.

VOCABULARY

An array is an arrangement of objects in rows and columns.





4. Explain, ***So in this activity when we increased the number of pips to double the product, were we adding another group? No. What was happening instead that caused the product to double?***

Assessment

Show or explain how you would solve this problem using Grasplify:

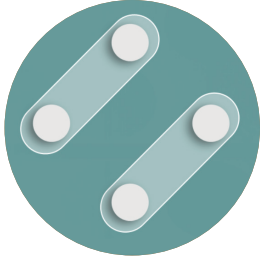
There are six people sharing a snack. Each person has two strawberries. How many strawberries would there be in total if you doubled the number of strawberries each person got?

Resource Connections

In the Grade 4 textbook, Math Makes Sense (p. 82–83), doubling is a strategy to multiply using arrays. To introduce students to arrays, have them explore the "array" button in Grasplify (top right corner). The calculation of 4 x 6 can be completed by creating an array of 2 x 6 and joining another 2 x 6 array below it (doubling). Noting that the 6 in the new array remains the same, but the 2 is doubled, can be a nice connection to the Grasplify tasks above, which show that to double the product, you only need to double one side (factor) of Grasplify. Of course, you can also double the array so that the 2 remains the same and the 6 is doubled (recreating the array to the right of the original rather than below), to support the idea that only one factor needs to be doubled. Use the array button in Grasplify to draw students' attention to doubling and halving during the activity above.

TEACHER TIP

This can be shown either by taking a screenshot or through drawing. Some students will do 6 x 2 = 12, since that number comes first in the problem, though it makes more sense to do 2 x 6 = 12 and then double the two to create 4 x 6 = 24.



Halving

Summary

Students create pictorial representations of multiplication using Grasplify to investigate the mathematical strategy of halving, which is more multiplicative in nature than repeated addition.

Video Link

Curriculum Objectives

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for this task.

Content

- Provide opportunities for concrete and pictorial representations of multiplication
- Using mental math strategies such as doubling or halving
- Fluency with math strategies (e.g. identifying related doubles)
- Understanding concepts of multiplication (e.g. arrays)
- Multiplication and division are related

Indigenous Connection

See the doubling, halving Indigenous connection previously outlined on page 17.

Introduction

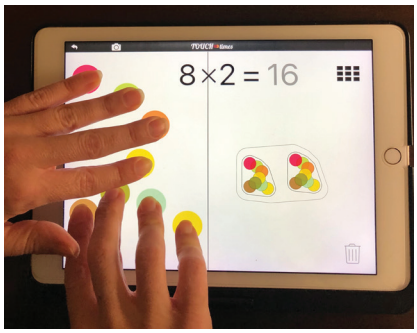
1. Tell students, ***Last class we started with a product of three and doubled it to make a product of 6, which we doubled to make twelve, and doubled again to make a product of twenty-four.*** Use Grasplify to make the product 1 × 2 = 2 so that all students can see it projected on the screen. Emphasise that this has been made using one pip and two pods. You may want to take a screenshot of this, so that you can physically demonstrate on the projected image with your fingers, the creation of one pip and two pods. ***Today I want you to double the product so that it is four, changing only the number of pips. Once you've done that, double it again to make eight. Double it again to make sixteen. What is the final equation that you will have?***

TEACHER TIP

Instead of starting from the left, you can make 2 = 2 x 1 by starting on the right side of the screen. It's a good idea to use both directions to help students see that the equal sign does not always have to be on the right. This also provides an opportunity to discuss the meaning of the equal sign with students.

2. Give pairs time to complete this task.

3. Once students have successfully doubled 1 × 2 = 2 to create products of 4, 8 and 16 by only changing the pips, gather them together for a class discussion. Be sure to have Grasplify projected for all to see.



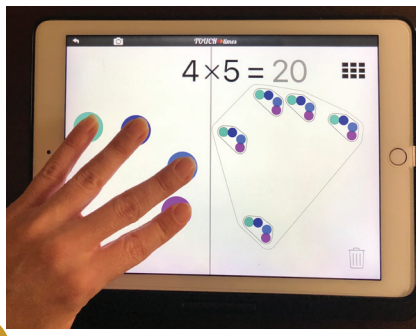
4. Have a pair demonstrate and explain each doubling step in the sequence leading up to the final equation.

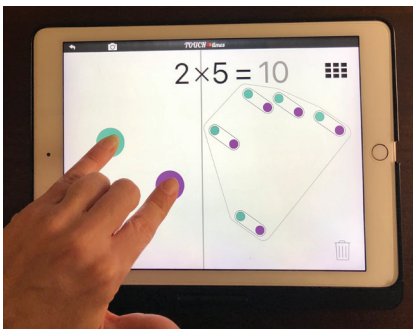
Extending Student Learning

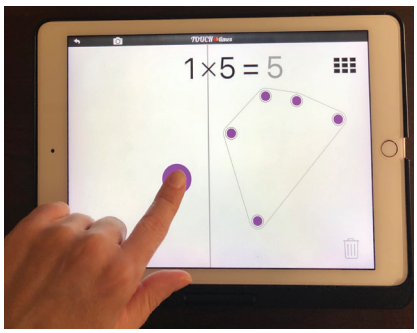
- More doubling variations can be given to early finishers.
- Invite students to triple the product. They will quickly run out of fingers, so it will be necessary to imagine how to triple the product and they can document their thinking on paper.
- In a move towards the more general, ask students if they had an unknown number of pips and 4 pods, what would they do to double?

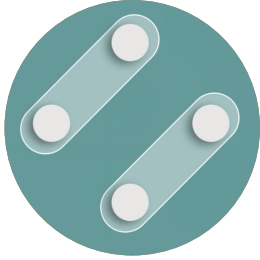
Task

1. Using Grasplify, make the product 4 × 5 = 20 so that all students can see it on the screen. ***Right now, the product is twenty, how can you halve the product so that it is ten? Your challenge is to halve the product so that it is ten by only changing the pips. Once you've done that, halve the product again to make five.*** Write the 20, 10, 5 sequence on the board for students to refer to while working. This is what the sequence of screens might look like.









2. As you circulate, observe and listen to students' conversations. Here are some things to watch for:

    - Students will have to shift from adding more pip fingers to removing half their pip-fingers each time.
    - Pay attention to the "choreography" students are using. For example, if a student places 2 fingers from her left hand as well as 2 fingers from her right hand on the pip-side, she can halve the product by lifting one hand. Similarly, if two students each have two fingers on the pip-side, they can halve the product if one of the students takes her fingers off.

WHAT TO WATCH FOR

Ensure that pairs have halved the product by changing the pips. When first given this task, many pairs halve the product by dragging pods to the trash.

Extending Student Learning

- Early finishers can be challenged to try halving the product of 8 × 5 = 40 by only changing the pips. Prompt them to try halving 40 = 5 x 8.
- Students can also be challenged to halve 10 × 5 = 50 which would quickly produce a problem (halving 25).
- Be sure to try an impossible question like 3 × 7, and ask about halving.
- In a move towards the more general, ask students if they had an unknown number of pips and 4 pods, what would they do to halve?

Summarise

1. Once the pairs have successfully halved 4 × 5 = 20 to create products of 10 and 5 by only changing the pips, gather them together for a class discussion. Be sure to have the Grasplify world projected for all to see.
2. Create 4 × 5 = 20 using Grasplify. You may want to leave your 5 pod-fingers as well as your 4 pip-fingers on the screen and pretend you do not know the answer to encourage students to explain. ***What do I have to do next in order to halve the product so that it becomes ten?*** Choose students to explain what you need to do. If they say lift 2 fingers, you may want to question, ***Which side?*** As they explain, ***Oh! The pods stay even if my fingers aren't on them. I forgot. Okay, so I lift two of my pip-fingers? Hey, that worked! How did you know that? Now what do I do?*** Continue to have students explain exactly what to do and question how they know.
3. Ask, ***How is doubling different from halving? Did you notice anything about halving that reminded you of what you learned***

about doubling last class?

Assessment

1. Rekha has made 32 = 4 x 8 in Grasplify. Show or explain two different ways she could halve that product to get 16.

2. Draw a picture or explain in words why halving is the opposite of doubling.

Resource Connections

If children are shown an array of counters, such as 6 × 4, they can explore various ways of "breaking up" the array. For example, children can be asked if 6 × 4 can be divided up into two equal arrays. In this case, 6 × 4 can be separated into 3 × 4 twice OR 6 × 2 twice. In each case, the halving of one of the factors can be highlighted. For example, 6 × 4 can be shown to be 3 × 4 × 2 or this can be tied to the doubling activity above. If children are asked to multiply 8 × 6, first they can halve 8 to 4, halve again to 2, multiply 2 × 6 and then double twice. The grade 4 Math Makes Sense textbook (p. 83), uses this method as a strategy to multiplying by 8.

In the Focus textbook (by Nelson), one of the questions posed that relates multiplication to division is: 20 dancers are dancing in equal groups. Show possible solutions using arrays. Connect halving and doubling as strategies to solve this problem.

The general idea in this resource connection is to tie the number of fingers that are touching the pip-side, with the number of vertical columns in the array. Using the array button in Grasplify can help to make this connection.

Many-to-One – Part A

Summary (for Part A & B)

The process of multiplication involves two stages of **unitisation** (combining many parts into one whole) to get a product. Students investigate how the placement of their fingers when using Grasplify can create a many-to-one unit. Students are meant to discover that *n* pips must first be established in order for a single *n*-pod to be created. For example, in the photo to the right, four pips are established and then five 4-pods are created with each touch.

Curriculum Objectives (for Part A & B)

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for these tasks.

Content

- Provide opportunities for concrete and pictorial representations of multiplication
- Looking for patterns in numbers to further develop understanding of multiplication computation

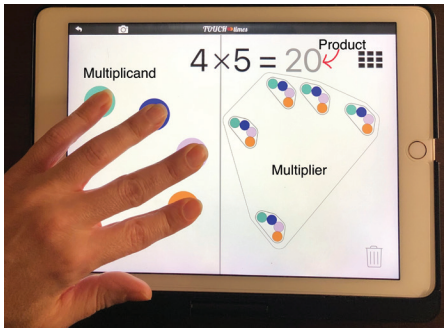
Indigenous Connection

Unitising

A folding algorithm that results in an odd number of units on a folded piece of paper has been developed by some Indigenous nations. By folding a strip of paper in half multiple times will only divide the strip of paper into units of powers of 2. To create an odd number of units on a strip of paper (or any foldable material). A knowledge holder would lightly fold a short section at one end, estimating the length of the unit. They would then lightly fold the remaining strip into halves multiple times to match the original unit. Once the original unit and the strip of halves are equal in length, the folds can be creased. The final result is a strip with an odd number of units.

Cultural Connection

Video Link

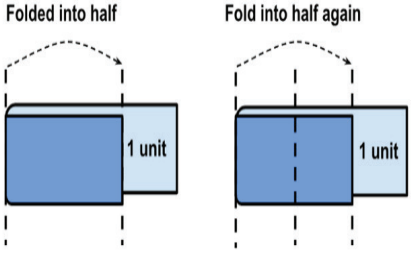


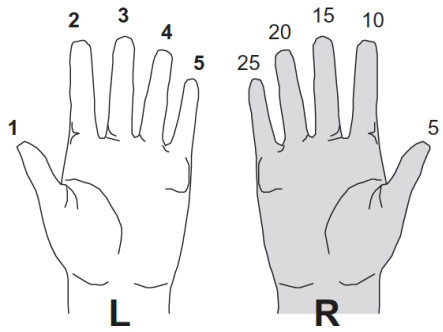
VOCABULARY

The multiplicand (group size) is represented by the pips.

The multiplier (number of groups) is represented by the pods.

The product is the result of multiplying the multiplicand by the multiplier.



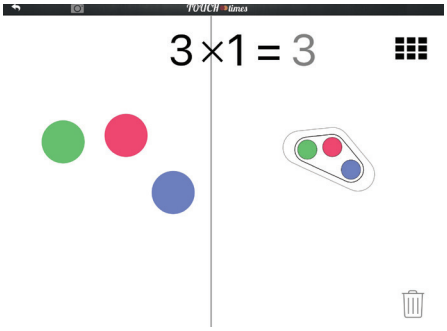


In almost every culture that has been documented, people have used their fingers, as well as other body parts, to count and to do arithmetic. We are accustomed to counting on our fingers and assigning each finger a digit from 1 to 10. In India and China, however, people historically counted up to five on one hand and then used the fingers of the other hand to count by fives (as in the picture on the left) or to represent powers of 10. In the design of Grasplify, the fingers of the pod-hand are counting by the quantity created by the fingers of the pip-hand: when 5 pips are made with the pip-hand, these five pips become 1 pod when a finger from the pod-hand is placed on the screen. This process of 5 becoming 1 is fundamental to multiplication.

Introduction

1. Allow a short period of time for students (alone or in partners) to explore the app. Assist anyone experiencing technical difficulties.

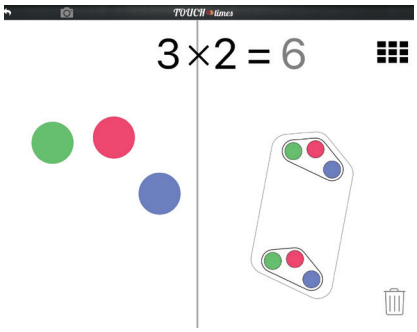
2. Gather students together and display the image on the left. Explain that, ***I have one pod of three, how can we make two pods of three?*** Give students some time to experiment with this. Once students have successfully created 2 pods, have a pair demonstrate and explain how to make 2 pods of three. Ask, ***Can anyone explain how we doubled here?*** Be explicit in pointing out that we doubled be creating another pod. Ask, ***How could you double the product using pips?*** Again, give children time to experiment with this idea of doubling the pips rather than the pods. Once students have done this, have a pair demonstrate and explain how to double the product using pips. Ask, ***How is doubling with the pips different than doubling the pods?*** This is an opportunity to remind children of how the pips **spread** across the pods.

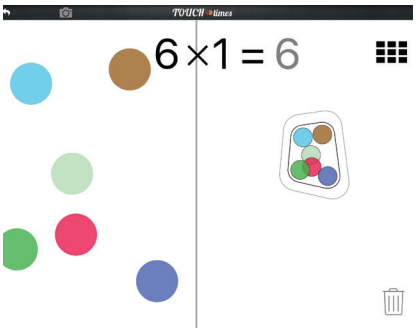


TEACHER TIP

An important aspect of Grasplify involves focusing children's attention on changing the number of pips, which emphasises the **spreading** aspect of multiplication. This is very different than repeated addition.

Task

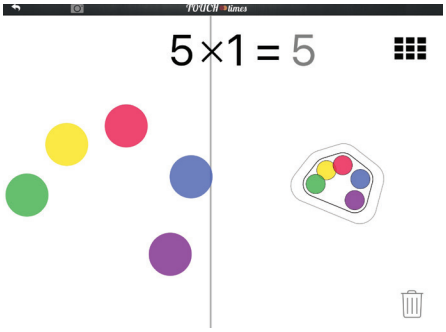




1. Have students complete this task with a partner in order to encourage the sharing of ideas and to allow for the possibility of more fingers to press or tap the screen at the same time.

2. Tell students, ***With your partner, your task is to figure out how to make a single pod of five. A single pod of five can also be called a 5-pod and will look like this.***

3. As you circulate, observe and monitor students' conversations. Here



are some things to watch for:

- Students must generate 5 pips first in order to create a single pod of 5 with one touch.
- It is common for students to initially place 1 pip-finger down and then tap 5 times sequentially with a pod-finger making five 1-pods instead of one 5-pod (see Figure a).
- If this strategy persists, invite children to try placing several pip-fingers on the screen simultaneously.
- Depending on which side of the screen students tap first, their number sentence may reflect 5 × 1 = 5 or 5 = 1 × 5 (Figure b). Either of which is a successful creation of a 5-pod.

CAUTION!

The many-to-one concept appears easy but children do not initially find this easy to do or easy to understand.

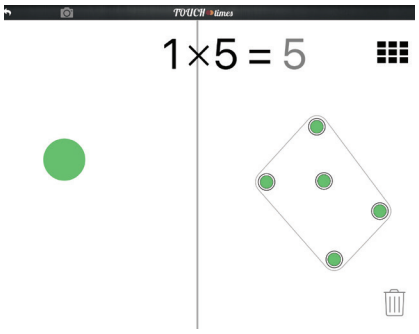


Figure a: One-to-Many
Five 1-pod finger touches

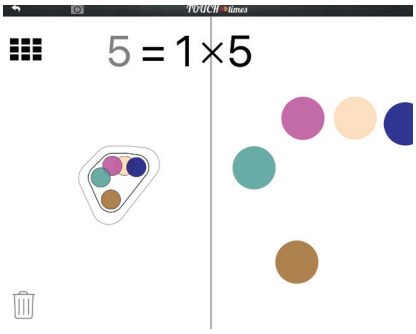


Figure b: Many-to-one
One 5-pod finger touch

Extending Student Learning

As pairs successfully create a pod of 5 with one pod-finger, draw their attention to the many-to-one concept, ***You made five with one pod-finger!*** This highlights the process of 5 becoming 1, which doesn't typically come out in our way of talking about multiplication. Direct their attention to the colours by asking, ***You have five, what do you notice about the colours in the pod?*** You can then push them further by asking questions such as, ***How can you make three with one pod-finger? How can you create a single pod of seven? What is the biggest number that you can make?***

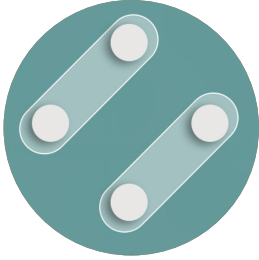
TEACHER TIP

- Older generation iPads will allow a maximum of 11 pips.
- Newer generation iPads will allow a maximum of 17 pips.

4. As you circulate, observe and listen to students' conversations. Here are some things to watch for:
    - The goal of this task is for students to develop fluency in performing the **unitising** action with different numbers, using only 1 pod-finger.
    - If they don't immediately create the pod of 5 or 7 with one pod-finger, prompt them to do so.

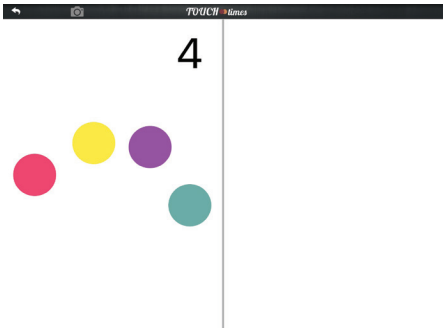
Summarise

1. Once the pairs have successfully created a single pod of 5, gather them together for a class discussion. Be sure to have the Grasplify world projected for all to see.
2. Ask for a pair to demonstrate and explain to the group how they created a single pod of 5. Encourage the pair to verbalise what they did. Prompt students to elaborate upon their explanation with questions such as, ***Did you get this right away? If not, what did you do first? How did you know that was incorrect? How did you figure out how to create a 5-pod instead of five 1-pods? I kind of felt powerful when I did this and was able to put down ONE pod-finger to make a five, did anyone else feel powerful when they did that?***
3. While a pair is demonstrating their five pips and one pod, ask students to predict, ***What will happen if we put one more pip down?*** After a few predictions are shared, have the pair add a pip so that all can see what happens. ***What happened to the shape of the pod? What do you notice about the colours? If you put one pip down and then take one off, what will happen to the colour and the shape?*** An important part of multiplication in the Grasplify world involves seeing how the colours work (**spreading**) and the various aspects of the co-ordination of quantities (**unitising** and **multi-plying**) that relate to both colour and shape.



4. Ask, ***Did anyone else find a different way of doing this?*** This provides an opportunity to discuss and demonstrate how 5 × 1 = 5 and 5 = 1 × 5 are both ways of creating a single 5-pod. ***Does the order of the number sentence in this case matter? Why or why not?*** Make explicit that the 5 (multiplicand) represents the 5 pips and the 1 (multiplier) represents 1 pod, which in turn produces 5 (product). The mathematical terminology is not necessary, but the ideas represented by the pips and pods in this case are important ideas for thinking multiplicatively.

Assessment



1. Using the picture on the right, draw what the right half of the screen would look like if you were to place one finger there.

2. Show how to make 1 bag of 6 marbles in Grasplify.

Resource Connections

Cuisenaire Rods – This many-to-one activity can be replicated using Cuisenaire rods. ***Can you find five of the same units that when put together will be the same as one rod?*** There will be more than one way to show this relationship. You may want children to share with the group the different ways that this can be done. This could be projected using a document camera so that the size and colour of the units and rods can be seen or students can be gathered in a circle on the carpet and each idea can be explained and displayed in the centre of the circle for all to see. ***So, five white units made one yellow rod and five green units made one orange rod,*** (colours may vary depending on the Cuisenaire rods that your school has) ***how are these the same? How are they different?*** We want students to understand that 5 units can be transformed into 1 rod (many-to-one). As long as all 5 units are the same size, it doesn't matter how big the units are as long as 5 of them correspond to a single rod.

Straws – This many-to-one activity can also be replicated using straws, popsicle sticks or other manipulatives that are easily available. ***Can you figure out how many straws there are by placing them into bundles or groups?*** Here the first action is to make the groups, which involves determining the unit, and then to find out how many groups there are. This type of activity is often done during morning calendar but associated with place value. This many-to-one idea is also an important skill for thinking multiplicatively.

Many-to-One – Part B

Video Link

TEACHER TIP

We want students to develop fluency in performing the unitising action with different numbers, using only 1 pod-finger.

TEACHER TIP

- Older generation iPads will allow a maximum of 11 pips.
- Newer generation iPads will allow a maximum of 17 pips.

WHAT TO WATCH FOR

Do students pay attention to the shape and colours of the pods?

Introduction

1. Tell students, ***Yesterday we learned how to make a five with one pod-finger. With your partner, I'd like you to explore what other numbers you can create with only one pod-finger.*** Provide time for students to do this. If they don't immediately create another pod such as 7 with one pod-finger, prompt them to do so.

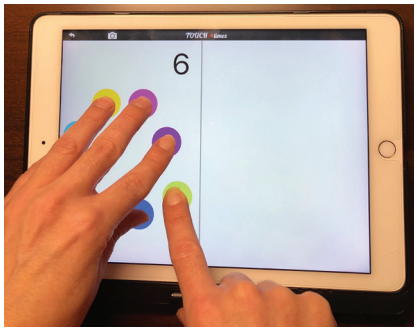
2. Bring the class together and have pairs share with the class the numbers they have created with one pod-finger. Given this introductory task is only a slight extension of what was done in the previous lesson, this is a good opportunity to have students who often don't get a chance to share new ideas with the group, come to the iPad to share their discoveries.

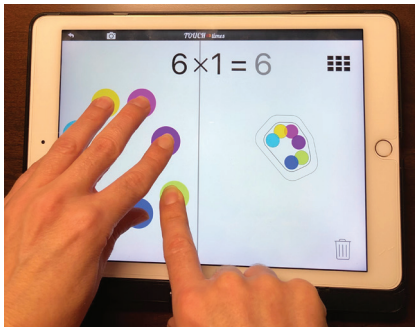
Task

1. Tell students, ***Now that you are able to make many different numbers with one pod-finger, your next challenge is to figure out what is the biggest pod that you can make? I am also going to ask you to explain how you know that it is the biggest pod that you can make.*** The point of this task is for students to discover the limits of the iPad and is not specifically mathematical.

2. Gather students in front of the screen where Grasplify is projected for all to see. Choose a pair to demonstrate the biggest pod that they could make and explain how they know it is the biggest pod possible.

3. Students will complete the next activity with pencil crayons and paper. Explain that, ***Lucas puts six pip-fingers down on the screen, like this*** (project a screenshot—like the one below—that contains 6 colours you have available so that you can draw and colour in the pods in the summary part of this lesson). ***If his partner Benedict puts one pod-finger down to create a pod, draw what their pod will look like?***





Extending Student Learning

Early finishers can be encouraged to go further by asking them to ***Draw what would happen if Lucas lifted a pip-finger off the iPad? What if Lucas placed two more pip-fingers on the iPad?***

Summarise

1. After students have finished their drawings, project the image for all to see. Tape a large piece of white paper where the Grasplify image is being projected. Draw a vertical line on the paper to indicate the centre line in the middle of the iPad screen. Ask students, ***What will Benedict's pod look like? Describe it for me in detail so that I can draw it on the white paper.*** If necessary, prompt students for information about the shape of the pod as well as the colours of the pips. Ask, ***Does it have to be in this exact configuration or shape? Why? Do the colours matter? How do you know this pip (choose one) has to be this colour?***

2. Follow up with a hypothetical question, ***What if Lucas added another pip, how would the pod change?*** This is a good time to consolidate the vocabulary of pips and pods.

3. Once students have made their predictions about how the pod would change, demonstrate on the iPad. Ask them, ***Was your prediction correct? If not, what have you noticed that you didn't know before?***

TEACHER TIP

We want students to notice not only the **unitisation** but also the relationship between the pips on one side and the colour and shape of the pod configuration on the other side, as noticing this relation is crucial for seeing the **multi-plying.**

Assessment

Benedict had created the pips and pods that you can see in the photo on the right. Draw a picture showing what the screen would look like after Benedict places two more fingers on the right side of the screen.

Resource Connections

Giving students both regular and irregular shapes to find area using tiles can support the many-to-one metaphor. Using shapes on grid paper, dot paper or using an elastic band on a geoboard, students can determine how many square tiles are needed to cover or fill in the shape. The focus is on area, not on calculation. Once the tiles have been laid and counted, the area (unit) is expressed using a multitude (how many tiles). This is an example of strengthening student's understanding of **unitising**. The area is a unit of many.

TOUCH times

Skip Counting by Pips

Video Link

Summary

Students explore the effects of changing the number of pips on the composition of all of the pods using Grasplify. This more transformational approach to multiplication focuses on how each pip **spreads** across every pod.

Curriculum Objectives

Big Ideas & Curricular Competencies

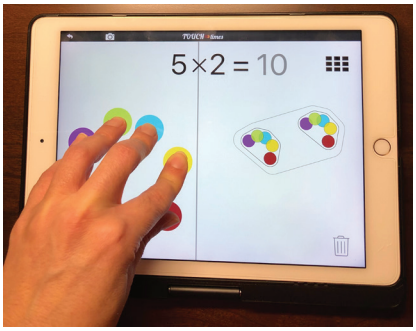
See page 11 for the Big Ideas and the Curricular Competencies for this task.

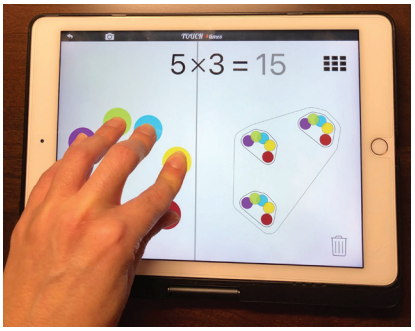
Content

- Connect multiplication to skip counting
- Connect multiplication to division and repeated addition

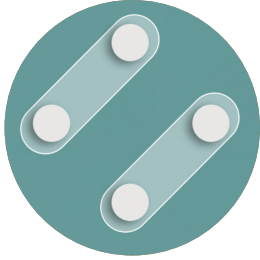
Exploration

1. Project Grasplify for the class to see. Tell students, ***Last class you made a single pod of five by placing five fingers on the pip-side and one finger on the pod-side.*** Demonstrate what it looks like when the number of pips are changed by placing down or lifting fingers on the pip-side. Say, ***I'm changing the number of pips.*** Then place one finger at a time on the pod-side to generate the products 5, 10, 15, 20 and say, ***Now, I'm changing the number of pods.*** Students will likely recognise the skip counting. You might do more skip counting using a different number of pips. Say, ***I've shown you how to skip count by five by changing the number of pods. Now your job is to find a way to skip count by five by changing the number of pips.***





2. Allow time for students (alone or in partners) to work on the task.



3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:

    - Ensure that students are skip counting by changing the pips. When first given this task, many pairs will skip count by adding more pods (see previous pictures), which is similar to repeated addition.
    - A hint might be to tell students to make 5 a different way than 5 pips and 1 pod.
    - Skip counting by changing the pips requires students to first place 1 pip-finger down and then create 5 pods for a product of 5. Students then place additional pip-fingers down, one at a time.

Extending Student Learning

- Early finishers can be invited to try skip counting by 2 or 3 or 10.
- Invite students to skip count by a larger number that they are not as familiar with and to record the products in their notebooks.

Summarise

1. Once students have had an opportunity to solve the task, gather them together for a class discussion. Project Grasplify on a screen, so that it is available for students to demonstrate and explain their discoveries to their classmates. ***What did you notice about Grasplify? How is skip counting by pips different than skip counting by pods?***

Extending Student Learning

It is also interesting to explore skip counting backwards. When asked to do this, students will be tempted to make the product and then remove the pods by dragging them into the trash. Though a valid way of skip counting backwards, this emphasises the repeated addition (or subtraction) model of multiplication. So be sure to encourage students to skip count backwards by changing the number of pips.

A first question can be, ***Make a product of twenty-five in Grasplify and skip count backwards by five by changing the number of pips.*** This should be relatively straight forward since 25 is a square number. By providing different initial products, however, the task will become more challenging. For example, you can give the following task: ***Make a product of fifteen in Grasplify and skip count backwards by three by changing the number of pips.*** Students will first need to decide how to make the product—3 pips, 5 times, or the other way around.

TEACHER TIP

Students are engaging in the process of multi-plying here, since they are seeing each new pod as a copy of the unit of pips. The pip-configuration stays the same which isn't something easy to demonstrate with physical objects.

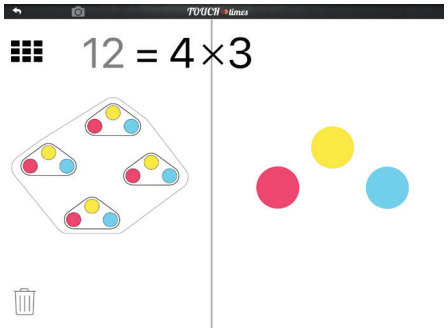
CAUTION!

The choice of the factors that go into the product matters for this task.

Even more challenging would be tasks of this form: ***Make a product of twenty-four in Grasplify and skip count backwards until you reach twelve by changing the number of pips.*** This can be done by counting backwards by twelves, starting with 2 pips and 12 pods; but it could also be done by counting backwards by sixes, starting with 4 pips and 6 pods.

Assessment

1. Sanna did some skip counting by changing the number of pips using Grasplify. She made this sequence of products: 3, 6, 9, 12, 15, 12, 9, 6, 3. Show or explain how she did this. You can use drawings, words or a combination of both to do this.



2. As a formative assessment task, project the image to the left.

Ask students to answer the following questions:

- Sanna used Grasplify to skip count to 12. Describe each of the steps that she did if she was skip counting by changing the pips. If students need a hint, ***Sanna started with just one pip.***
- Imagine that Sanna placed another finger on the pip-side of the screen. Draw what we would see on her screen.

Resource Connections

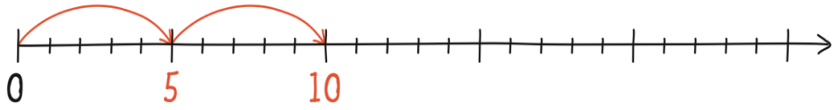
A common problem given to students that requires skip counting is: ***There are three wheels on a tricycle. How many total wheels are there when there are five tricycles?***

Linking cubes can be used to model this situation. Students create groups of 3 cubes for each tricycle. They make 5 groups of 3 and count them to find the answer. In Grasplify, they can model this problem by pressing 3 pip-fingers on one side and 5 sequential touches on the pod-side, so the answer is 15. Grasplify allows the opportunity to change tricycle to bicycle (change 3 to 2), or tricycle to car (change 3 to 4) by simply changing the pips. This can also be modeled using linking cubes by adding or removing a cube in each group. Grasplify offers a more dynamic and fluent process to draw attention to the **spread** of pips throughout each of the pods.

Give students 30 counters and have them count by 1s, then by 2s, 3s, 5s, 6s and 10s. They should notice that it doesn't matter what they skip count by, the final count is always the same (30). They should also notice that when the size of the count, the unit, is bigger, there are fewer groups. This connects to skip counting by pips because when the pips are increased by 1, the pods will all grow by 1, showing that we would need fewer pods to

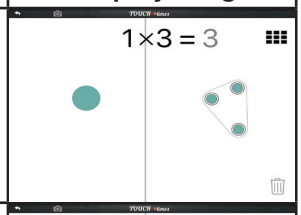
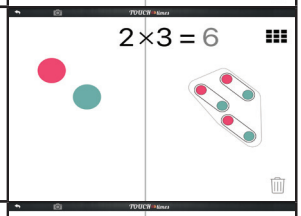
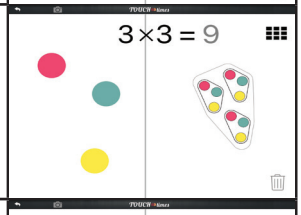
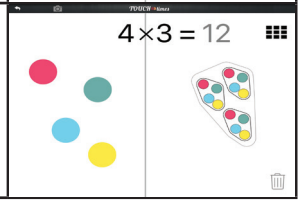
count to a particular value.

The Math Makes Sense (grade 4, p. 91) or Focus (p. 188–189) textbooks show how to use skip counting on a number line to model this situation. ***A large paper clip is five centimeters long, how long is a line of four paper clips?*** Skip counting on a number line can be used to show the connection. A follow-up question could be, ***What if the paper clip was***



six centimeters long? How could we model this on a number line?

Tables – In this activity, students explore changing the size of units (changing the multiplicand), which can be paired with the use of a table. The table highlights the functional view to multiplication—in this case, the function is to triple. The first example in the table below shows that the relation between the number of pips (1), the product (3) and the number of pods is 3 (as seen in the Grasplify image). This table can be extended by changing the number of pips. The number of pips has increased by 1 in each row. The functional relation is being emphasised because it shows that if 1 pip tripled makes 3, then 4 pips tripled makes 12.

Number of Pips	Product	Grasplify Image
1	3	
2	6	
3	9	
4	12	

Pips versus Pods

Video Link

Summary

This task invites students to attend more closely to the different roles that the pips and the pods play in multiplication.

Curriculum Objectives

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for this task.

Content

- Provide opportunities for concrete and pictorial representations of multiplication
- Looking for patterns in numbers to further develop understanding of multiplication computation

Introduction

1. Project Grasplify onto a screen for the class to view. Tell students, ***Last class we looked at skip counting by fives using Grasplify. Today your job is to come up with different ways to skip count by threes.***
2. Allow time for students to work on the task. Be sure to encourage students to find more than one way to skip count by threes.
3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:
    - Ensure that students skip count by changing the number of pips, as well as skip counting by changing the number of pods.

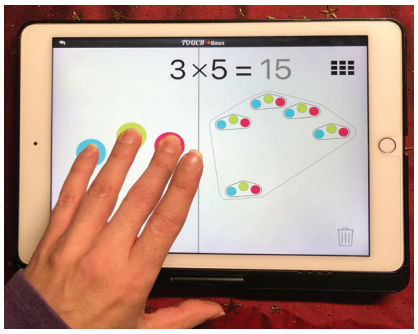
Extending Student Learning

- Early finishers can be challenged with, ***You've shown me different ways of skip counting by going up by threes, how can you skip count down by threes?***

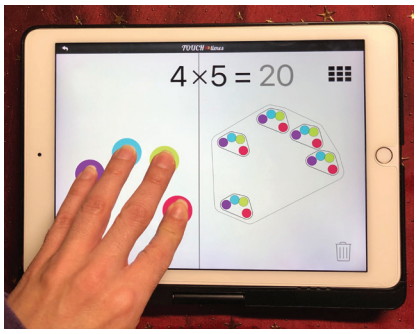
4. After a short period of exploration time, bring the class together to review the task. Project Grasplify on a screen, so that it is available for students to demonstrate and explain their discoveries to their classmates. Ask, ***What are the different ways of skip counting by three using Grasplify? How is skip counting by pips different than skip counting by pods? How is skip counting up different than skip counting down? How are skip counting up or down similar?***

Task

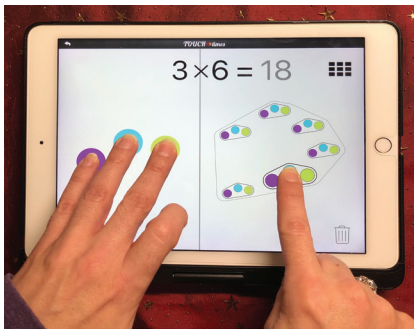
1. Have pairs seated together with a shared iPad so that they can work through these tasks as you give them. Say, ***Make 3 x 5 = 15 on your iPad. Your partner can only put one more finger on the screen. Where should it go to make the larger product?***



Begin with 3 x 5 = 15



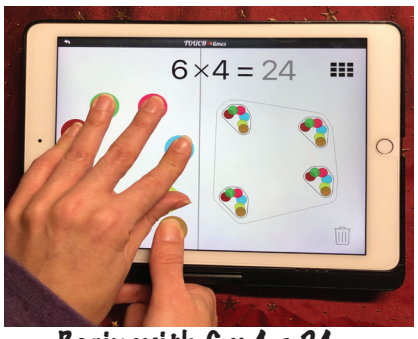
Add a pip-finger or



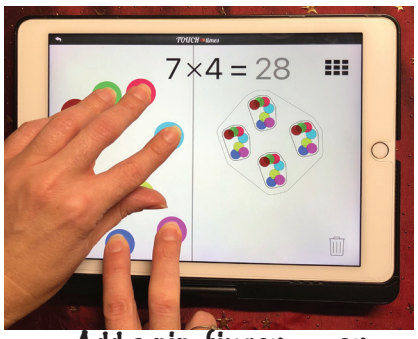
Add a pod-finger

2. Give the pairs time to explore this idea. We want students to notice the relationship between the pips and the pods and be able to explain how the product is influenced by the addition of a pip or a pod. By adding a pip to 3 × 5 = 15, it increases the product by 5 because an additional pip appears in each of the existing 5 pods. Alternately, by adding a pod, it increases the product only by 3 because an additional pod includes 3 pips.

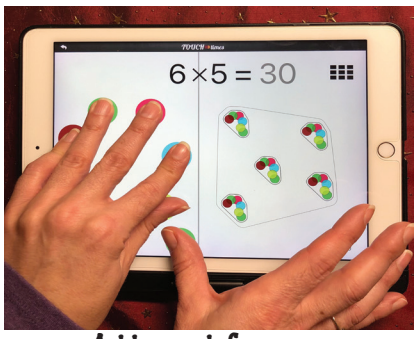
3. Once the pairs have determined an answer, challenge them to make 6 x 4 = 24. ***Where should the extra finger go now to make the larger product?***



Begin with 6 x 4 = 24



Add a pip-finger or



Add a pod-finger

4. Give time to explore this idea. Bring the class together for a discussion. Project Grasplify on a screen so that students can demonstrate and explain their discoveries to their classmates. Choose a pair to demonstrate and explain whether they would add a pip-finger or a pod-finger to 3 × 5 = 15 to create the largest possible product. ***What happens to the product when we add a pip-finger? What happens to the product when we add a pod-finger? Which creates the larger product? Why?*** Repeat this process for 6 × 4 = 24.

5. Ask, ***Predict how to make the larger product for 5 x 5, 2 x 6, 6 x 2. Would we add another pip or another pod? Turn and talk to the person beside you about your prediction.*** Once students have shared their predictions, ask a student to create 5 × 5 on the iPad. Then have the student place an additional pip-finger on the screen. Ask, ***How did that extra pip affect the product?*** Have the student remove the extra pip finger and place an additional pod-finger on the screen. Ask, ***How did that extra pod affect the product? So, which created the larger product for 5 x 5, another pip or another pod?*** Do this for each of the remaining equations.

6. Ask, ***Does anyone know that their prediction will be correct for sure, without trying it out on TouchTimes? How do you know?*** Try to generate a more general statement, such as, "*You can make a bigger product by putting the extra finger on the side with the smaller number.*" Have students explain why this works. Ask, ***What do you do when both numbers are the same? How do you know which will create the larger product?***

TEACHER TIP

You may want to record on the board this reminder.
3 Pictures:
1) 3 x 4 = 12
2) How 3 x 4 = 12 changes with 1 more pip
3) How 3 x 4 = 12 changes with 1 more pod

Summarise

1. Explain, ***In your notebooks, draw what 3 x 4 = 12 would look like in Grasplify. Please use your pencil crayons to show what the colours would look like in the pips and the pods. Then draw a second picture that shows what Grasplify would look like if you added another pip-finger. Finally, draw a third picture that shows what Grasplify would look like if you had 3 x 4 = 12 and created another pod.***

Assessment

Alternatively, invite students to come up with a context that models the pips versus pods situation they have explored. For example, if there are 4 motorcycles, each of which has 3 wheels, would you have more wheels in total by having an extra motorcycle or by having an extra wheel on each motorcycle? This might initiate a discussion about different circumstances in which it would be better to have more wheels or less wheels. A picture of a 3-wheeled motorcycle may help some students.

Rearranging Pods

Summary

In this task students explore using the disembedding method to find different multiplication number sentence combinations. When using the disembedding method, students are breaking the whole into parts that are easier to manipulate or think about and then putting them back together into a whole to determine the product. For example, one way of disembedding 8 x 6 could be 8 x 5 = 40 and 8 x 1 = 8, which makes a product of 48.

Video Link

Curriculum Objectives

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for this task.

Content

• Provide opportunities for concrete and pictorial representations of multiplication

• Connect multiplication to skip counting

• Looking for patterns in numbers to further develop understanding of multiplication computation

Introduction

For this task, make sure that the Numeric Factor Values and the Numeric Product Values are turned off in the Settings. The Settings button can be found in the bottom left hand corner of the main *TouchTimes* screen.

SETTINGS

Remember to turn off the Numeric Factor Values and the Numeric Product Values in settings.

1. Project Grasplify onto a screen for the class to view. Tell students, ***I bought some cases of pop at the store. There are 6 in each case and I bought 5 cases. Can someone come show me what that would look like using Grasplify?*** Choose a student to come up to model this situation on the projected iPad for all to see.

2. After the student has done this, ask him or her to arrange the 5 pods so that they are clumped together. Say, ***I had 30 bottles but then my daughter asked me to buy two more cases because she wanted to try two other flavours. How can I use Grasplify to model or show this scenario? And how many bottles will I now have in all?*** It will be helpful for students if you articulate and explicitly demonstrate this process.

- Some students may suggest that you remove the original 5 pods and make 2 pods instead. Create 6 x 5 for students to see. Remove your fingers to reset the screen and then create 6 x 2. Comment that, ***These are easy products to find, 30 and 12.***

- From 6 x 5, others may suggest placing 2 additional pods on the screen. Encourage them to set the 2 new pods slightly apart from the 5 existing ones. Ask, ***How would you describe this as a multiplication equation?*** Students will see this as 6 x 7. ***So, can I figure out 6 x 7 by first finding 6 x 5 and then finding 6 x 2? What do you think? When we break multiplication equations apart like this, it's called disembedding.***

3. Ask students, ***Are there other ways that we can figure out 6 x 7 by using this disembedding method? Let's use this idea to figure out some other products.*** Students might suggest 6 x 4 and 6 x 3 or 6 x 2 and 6 x 2 and 6 x 2 and 6 x 1. Have students explain their ideas so that you can model them on Grasplify for everyone to see. Students might also suggest disembedding the pips, so doing 3 x 7 first and then placing three more pip-fingers down afterwards to create 6 x 7.

Task

1. Explain to students, ***Your task today is to use the disembedding method to create the following number sentences in Grasplify, using combinations of other number sentences that you or your partner already know. Be sure to record your number sentence combinations on your mini whiteboard so that you can keep track of how many different combinations that you discover.***

6 x 8

9 x 13

17 x 11

13 x 21

TEACHER TIP

Make sure that students group the sets of pods that they create and that they take a screenshot of their solution, and annotate on it to show their embedding.

TEACHER TIP

Students can turn the Numeric Product Values back on to verify their solutions.

Extending Student Learning

- Once pairs who have successfully found different number sentence combinations for each of the given number sentences, challenge them to figure out the product for each.

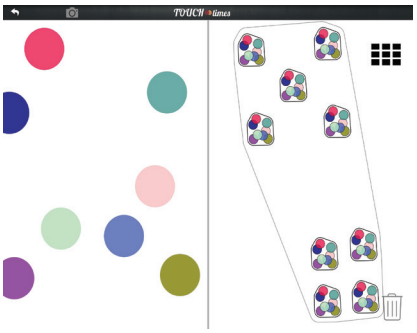
- Early finishers can be invited to make up a set of factors of their own choosing.

Summarise

Once students have had an opportunity to solve the task, gather them together for a class discussion. Project Grasplify on a screen, so that it is available for students to demonstrate and explain their solution methods to their classmates. Encourage them to discuss why they like certain disembeddings more than others. For example, with respect to the example given in the introduction, some students might find 6 x 2 and 6 x 2 and 6 x 2 and 6 x 1 easier because 6 x 2 involves doubling, which is easier to remember, but others might say it takes too long to add up all of the components. Some might prefer disembeddings that use 5 or 10 since they involve products that are easy to find.

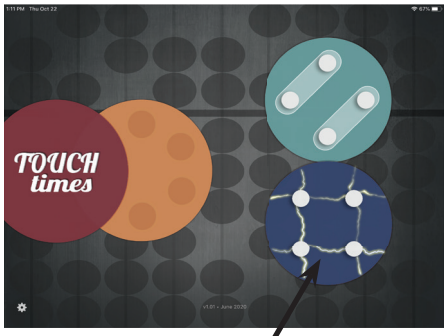
Assessment

Tasha took this screenshot. Write down the multiplication equation and explain how she used disembedding to find the product.



ZAPLIFY EXPLORATION

Exploration



1. Project the *TouchTimes* homepage onto a screen for the class to view. Tell students, ***Today you're going to use an app called*** **TouchTimes.** ***When you get your iPad, you can press this button (indicate which button to push) to open the Zaplify world. You will then have time to play with and explore the app. While you're exploring, pay attention to what is happening in Zaplify and jot down what you notice or things that you wonder.*** Mini-whiteboards are a great resource for students to do this.

2. Allow time for students (alone or in partners) to explore the app. While children explore the app two hands must be used, otherwise little happens. Assist anyone experiencing technical difficulties. Be sure students have opened the Zaplify world.

Vocabulary Alert! Watch for opportunities to introduce and reinforce the terms in the vocabulary box.

3. Ask questions to determine how students are thinking about the use of Zaplify. ***What happened when you placed your fingers down on the side first? What happened when you placed your fingers on the bottom first? What do you notice about the numerals on the side and at the top of the screen? How did you make that happen? Can you place multiple fingers down at once (on the side/on the bottom)? What happens when you do that? What things did you notice about the intersections?*** Encourage students to try different combinations of actions with their hands.

4. Once students have had an opportunity to get a feel for how Zaplify works, gather the class together for a whole group discussion. You may want to project Zaplify on a screen, so that it is available for students to demonstrate and explain their discoveries to their classmates. ***What did you notice about Zaplify?*** As students share their ideas, record these on an I Noticed/I Wonder chart for all to see. ***Is there anything that you wonder about Zaplify?*** Record these ideas on the chart as well. Use the questions above to encourage more discussion and demonstration from the class.

VOCABULARY

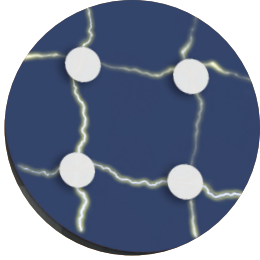
The number of lightning bolts from the side represent a factor.

The number of lightning bolts from the bottom represent the other factor.

The total number of intersection points represent the product.

TEACHER TIP

The term factor erases the separation between the multiplicand and the multiplier and assigns identical properties to each unit of count.

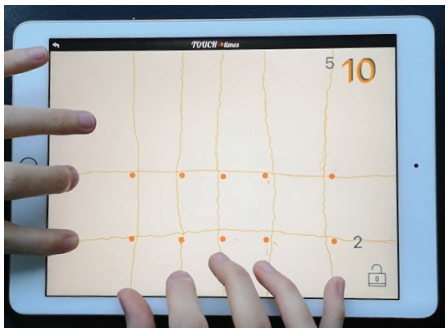


Multi-plying Factors

Summary

In Zaplify, all numbers represent magnitudes and the unit of the product derive from the multi-plying of the factors. Factors are represented by the number of lightning bolts, and the product is represented by the intersection points. The **covariation** between the factors and how it expresses the product is explored in this activity. For example, in the photo to the right, the product “ten” is established at the intersection of the horizontal and vertical lightning bolts.

Video Link



Curriculum Objectives

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for these tasks.

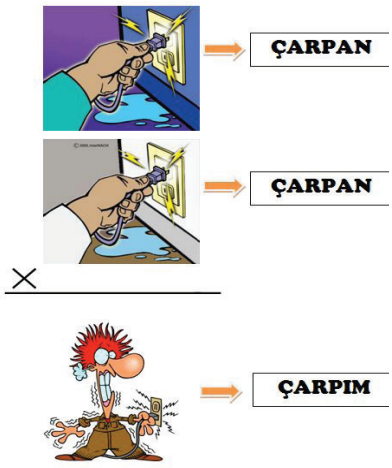
Content

- Provide opportunities for concrete and pictorial representations of multiplication

- Looking for patterns in numbers to further develop understanding of multiplication computation

Indigenous Connection

Some Indigenous nations use embodied and land-based conceptions to structure a counting system as well as spatial orientation. Conceiving of one’s body in relation to the ground upon which one stands, creates a vertical and horizontal connection, instantiating a binary of left/right. Thinking of a horizontal line at the waist, parallel with the ground, creates a notion of above and below. These crossing lines, the vertical body and the horizontal line through the waist, create an orthogonal structure as a frame of reference. In terms of number, counting can be associated with the connection between the body and the land through an embodied abacus, 1 is above and on the right, 2 is above and on the left, 3 is below and on the left and 4 is below and on the right. This orthogonal centre connects with the intersection of the lightning bolts.



Cultural Connection

Did you know that in some countries, two different words aren't used to designate the multiplier and the multiplicand? For example in Turkey, each factor is called "çarpan". The root of this Turkish term is also associated with electric shock in daily life. The image to the left is an example of a classroom poster used to introduce multiplication to students.

Connection to History

The intersection of perpendicular lines (orthogonal centre) has played a central role in Yupiaq daily activities that involve the use of symmetry and measuring, such as making snowshoes, mending nets and making ceremonial headdresses. Raphael Jimmy, a Yupiaq elder from Mountain Village, Alaska, enacts the orthogonal centre by raising two crossed fingers and describes it as "This is the center [qukaq] and beginning [ayagneq] of everything" (Lipka, Adams, Wong, Koester, Francois, 2019, p. 114).

French mathematician René Descartes formalised the Cartesian co-ordinate plane which consists of two axes perpendicular to each other. The axes are oriented horizontally and vertically, like Zaplify's lightning rods that emerge from the side and from the bottom. Using the axes of a Cartesian plane it is possible to index any point in the plane using an ordered pair of numbers. While the axes and the origin (the intersection points of the axes) are normally given to students when they first encounter the Cartesian plane, in Zaplify, students create their own Cartesian planes. This helps students develop an awareness of the structure of the Cartesian plane.

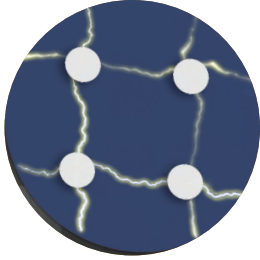
Introduction

Ask students to, ***Make a horizontal line on your iPad with your partner.*** Check to ensure that partners have done this correctly. ***Explain to me what I need to do to make a horizontal line.*** Following student directions, model doing this. ***Now I want you to make two horizontal lines on your iPad.*** Again, give students a few moments to do this. ***With your partner, while holding two horizontal lines, make a vertical line.*** Once students have done this, ask, ***Who can explain to me what I need to do to make a vertical line? What would I need to do to make three vertical lines?*** Ask students, ***What is the product? What are the factors making the product?***

VOCABULARY

The vertical lightning bolts go up and down.

The horizontal lightning bolts go side to side.



Task

1. Have students complete this task with a partner in order to encourage the sharing of ideas. Tell students, ***With your partner, your task is to make a product of ten. What is a product again?*** Ensure that all students understand what a product is. ***What colour will the product be on the screen? Your challenge is to figure out how to create a product of ten using Zaplify.***

2. As you circulate, observe and monitor students' conversations. Here are some things to watch for:

    - It is common for students to initially place ten fingers down to create a product of ten.
    - If this strategy persists, ask students, ***What is the product when you pressed ten fingers down? How could you make the product ten?***
    - Some students will consistently place their hands down in the same order, therefore always creating horizontal (or vertical) lines first. Invite children to explore how the order of the orientation of the lines influences the product.

TEACHER TIP

The product is the result of the intersection points created by the horizontal and vertical lightning bolts.

Extending Student Learning

- Encourage students to try making 12, try making 15, try making 1.
- As pairs successfully create a product of one, direct their attention to the relationship between the factors and the product, by asking, ***You made one with two fingers here. Can you make a single point right here?*** Point to a location on the screen that is different from where the previous lines intersected.

TEACHER TIP

Students may think that a point can be created only at a specific location. The goal is for students to figure out that a point can be created at any location as long as they create an intersection at that point.

Summarise

1. Once the pairs have successfully created a product of ten, gather everyone together for a class discussion. Be sure to have Zaplify projected for all to see.

2. Ask a pair to demonstrate and explain to the group how they created a product of ten. Encourage students to verbalise what they did. Prompt the pair to elaborate upon their explanation with questions such as, ***Did you get this right away? If not, what did you do first? How did you know that was incorrect? How did you figure out how to create a product of ten?***

3. Ask, ***Did anyone find a different way of doing this?*** This provides an opportunity to discuss and demonstrate how the order of the lines does not influence the size of the product. ***Does the order of the lines matter? Why or why not?*** Make explicit that the number of lines represent the factors, which are the black numerals on the screen, and the number of orange intersection points represent the product, which is the orange number at the top of the screen. The ideas represented by the lines and the intersection point in this case are important ideas for thinking multiplicatively.

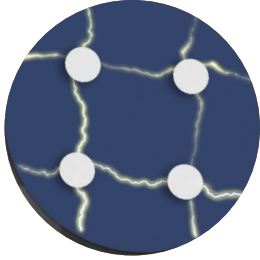
4. Ask a pair to demonstrate and explain to the group how they created a product of one. Ask, ***Was anyone surprised that you needed two lines in order to make a product of one?*** While the pair is demonstrating, ask students, ***What does the second line do to the first line? What does it create? Where is the point? When did it appear? If I want to make a single point right here*** (point to a location on the screen that is different from where the previous lines intersected), ***how should I place my fingers?***

TEACHER TIP

We want students to notice that in order to create an intersection point, they need two types of lines (vertical and horizontal). The number of intersection points is the product.

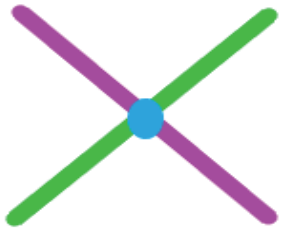
Assessment

If you press your fingers like the ones in the picture below, draw what you would see in Zaplify. Include as many details as possible and write the multiplication sentence that would go with this situation.

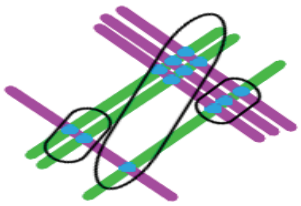


Resource Connections

1. This activity can be connected to what is sometimes referred to as the Japanese multiplication method. The product of multiplication can be determined by drawing perpendicular lines and counting the intersections. Similar to Zaplify, the lines are drawn in two different orientations which separate the factors spatially. You can also emphasise the distinction between the factors by asking students to use different colours. The number of green lines represents one factor, the number of purple lines represents the other factor and the number of blue dots represents the product. This picture shows 1 x 1=1.



2. This model can be used for multiplying multi-digit numbers in a faster way, as shown in the picture, which represents 21 x 13 = 273. Notice the 20 is represented by the two green lines with a larger space between the single green line, which represents the '1' in 21. When looking at the purple lines, they are also drawn to show the tens and the ones digits. Working from left to right, the first set of intersections on the left represents the hundreds digit in the product. The intersections found in the middle represent the tens digit in the product and the intersections on the right represent the ones digit.



TOUCH times

Order Doesn't Matter - Part A

Video Link

Summary

In Zaplify, the order of the factors doesn't matter and students can focus on how the product of two numbers is the same, whether they are made with 3 horizontal lines and 2 vertical ones or the other way around. In this task, students explore the commutative property of multiplication by creating the same product in two different ways and by comparing the configuration of the intersection points.

Curriculum Objectives (for Part A & B)

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for these tasks.

Content

- Using flexible amputation strategis (e.g. commutative principle)
- Understanding concepts of multiplication (e.g. arrays)

Introduction

Tell students, ***Today we will learn how to make the same product in two different ways.***

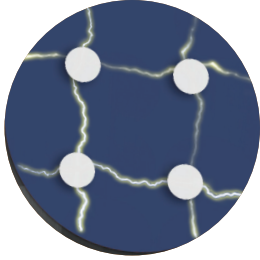
Task

1. Say, ***One partner will place two fingers along the left side and three on the bottom to produce six. The other partner will make a sketch of what Zaplify is showing.*** [Alternatively, students could take a screen shot.] ***Now, place three fingers along the left side and two fingers on the bottom. What did you produce? How is it similar to the sketch you made? How is it different?***
2. Allow time for students (alone or in partners) to work on the task.
3. As you circulate, observe and listen to student conversations. Here are some things to watch for:
    - Some students may only focus on the products. If this happens, prompt students to compare the general shape of the lines or the number of horizontal lines to the number of vertical lines sketches.

Extending Student Learning

TEACHER TIP

We want students to understand that the product of two factors will be the same, regardless of their horizontal or vertical placement.



Early finishers can be asked to make another product in two different ways. When students are asked such open-ended questions, some of them may not create the products based on the commutativity property. For example, they can produce 12 either as 3 x 4 or as 2 x 6. If this happens, direct them back to their drawings and ask them to compare how they produced 6 in two different ways and how they produced 12 in two different ways. Prompt them to come up with a rule to make their drawings the same. This might prompt them to rotate one of their sketches.

Summarise

1. Once the pairs have successfully finished making the products in two different ways, gather them together for a class discussion.

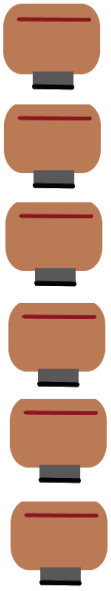
2. Have a pair to share their findings. Ask the pair, ***What is the new product? How does it compare to the sketch that you made? How many horizontal lines are in this sketch*** (point to their first sketch) ***and how many vertical lines are in this sketch*** (point to their second sketch)***?***

3. Make the product 6 by pressing six fingers along the side and one finger along the bottom and project this on the board. Ask students to compare the three shapes. Say, ***I made the product six in three different ways. Which sketch seems different than the other two? How is it different than the others?***

4. Explain, ***I see a relationship here. Do you see any connection between the first and the second sketch? What should I do to the second sketch so that it would look exactly like the first sketch?*** This question would prompt students to rotate their sketches. Model students' suggestions using Zaplify. Say, ***When we rotate the sketch, we change the orientation of the lines. Can you come up with a multiplication rule about the order of the factors?***

Assessment

The desks in Selin's classroom are organised in rows like the picture to the right. There are four rows of desks organised this way. One day Selin's class had to move to another room. However, the new room was smaller and could only fit four desks in each row. How many rows of desks will be in the new room? Draw how you can model this situation in Zaplify.



Resource Connections

This task can be related to the following multiplication table.

The products are symmetrically located in this type of multiplication table. When you draw a diagonal from the multiplication symbol towards the

X	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

product of 100, it can be seen that all the products are scattered around the table symmetrically with respect to that diagonal.

This table can be used in two ways:

1. Students can be given the table already filled in and asked to spot the same products in different parts of the table can colouring them with pencil crayons. You may provide the list of products and prompt students to use the same colour for the same products.

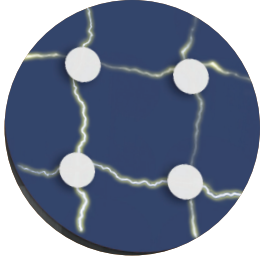
2. Students can be given an empty table and asked to fill it in by solving multiplication equations that are given in a list. Some examples include:

    - 1 x 2 & 2 x 1
    - 3 x 7 & 7 x 3
    - 6 x 9 & 9 x 6

Individual lists of numbers can be given according to your students' needs. If you want your students to practice a specific group of facts, you may include only these equations in your list.

Order Doesn't Matter – Part B

Summary



Video Link

Students continue to explore the idea of commutativity. Mathematical concepts frequently have multiple conceptualisations, each of which can be more suited to particular contexts—this is an important part of mathematics that may help students move away from the assumption that there is only one right answer or one right way of solving a problem.

Introduction

Say, ***Last class we made the same product in two different ways and came up with a multiplication rule. Could someone describe the rule?*** Listen to the students' explanations and restate the commutativity property of multiplication. ***This lesson we will try to solve a puzzle by using this rule.***

Task

1. Say, ***Claudia used ten fingers to produce twenty-one. Show two different ways she could have done this. Take a screenshot once you find a way to make twenty-one with exactly ten fingers.***
2. Allow time for students (alone or in partners) to work on the task.
3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:
    - Students are given a restriction in this task (they must use exactly ten fingers). This restriction limits the answer to two solutions (3 x 7 and 7 x 3). Otherwise, students might come up with 21 x 1 and 3 x 7 as two ways to make 21 and this would prevent them from exploring commutativity.
    - Some students may have difficulty getting started with this task. If that happens, ask them how they would produce 25. Once they have created 5 x 5, prompt them to change the product by changing the location of their fingers until they make the product 21.

Extending Student Learning

Early finishers can be asked to find as many ways as they can to make 24, draw sketches of each and identify which sketches look the same.

Summarise

1. Once the pairs have successfully finished making the products in two different ways, gather them together for a class discussion. Have a pair

TEACHER TIP

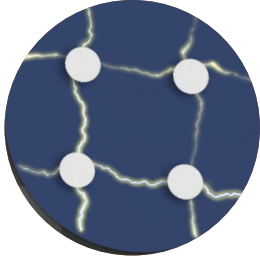
Shifting between the two photos on the screen might be difficult for students to compare the shapes. If the iPad is a newer version, you can display two photos at the same time on the screen. Otherwise, invite two students to place their iPads side-by-side to compare the images.

share their findings. Ask, ***Did you get this right away? If not, what did you do first? How did you know that was incorrect? How did you decide which factors to use to make the product twenty-one? How did it help you to figure out another way to make the product twenty-one? How does the configuration of intersection points of the first product compare to the configuration of intersection points of the second product?*** Wait for students' answers. Project the pair's photos on the board and ask, ***How many horizontal lines are in the first photo and how many vertical lines are in the second photo?***

2. Make the product 24 by pressing 6 fingers along the left side and 4 fingers along the bottom and project this on the board. Ask, ***Can you find another way to make twenty-four?*** Model students' suggestions in Zaplify and take a screenshot of each suggestion. Show the photos and ask, ***Which of these screenshots look the same?***

3. Explain, ***If you show me a product, I can quickly find another way to make that product with the help of the rule we learned last class. Can anyone tell me how that rule would help me make the same product in another way?***

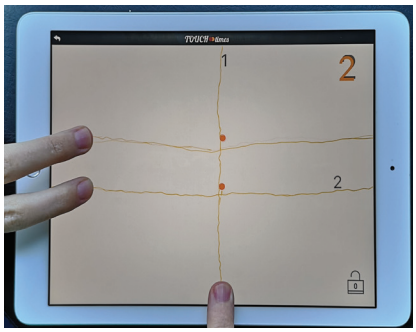
Unitising

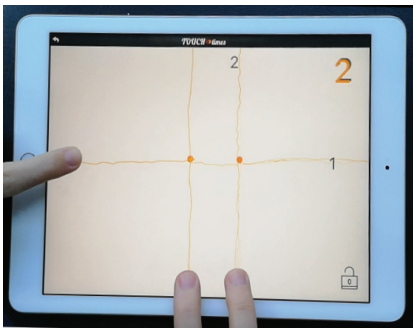


Summary

An important part of Zaplify involves seeing how the intersecting bolts of lightning work. For example, in the pictures below, two horizontal lines are established, and with the addition of one vertical line, a two-ple (two points on each vertical line) is created at the intersection points. Making two points at once highlights the process of **unitising** (two becoming one), which doesn't typically come out in traditional ways of talking about multiplication.

Video Link





Curriculum Objectives

Big Ideas & Curricular Competencies

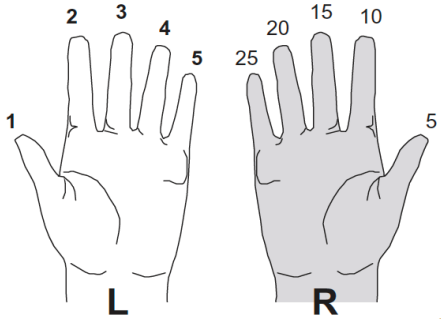
See page 11 for the Big Ideas and the Curricular Competencies for these tasks.

Content

- Provide opportunities for concrete and pictorial representations of multiplication
- Looking for patterns in numbers to further develop understanding of multiplication computation

Cultural Connection

In almost every culture that has been documented, people have used their fingers, as well as other body parts, to count and even to do arithmetic. We are accustomed to counting on our fingers from 1 to 10, where every finger gets assigned a single digit. But in India and China, people historically counted up to five on one hand and then used the fingers of the other hand to count by fives or to represent powers of 10 (as in the picture on the right). In the design of Zaplify, the fingers that create multi-ples (multiple points on a single line) are counting by the quantity created by the fingers of other hand. When five horizontal lines are made with one hand, these five lines



> create one five-ple when a finger is placed in the opposite side of the diagonal. This process of five becoming one is fundamental to multiplication.

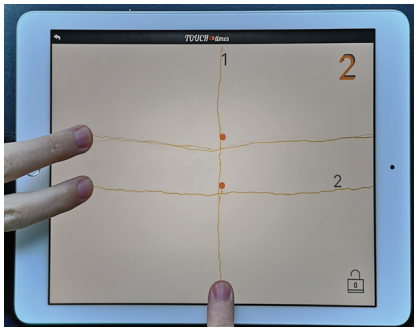
Introduction

Project Zaplify onto a screen for the class to view. Make one point and say, ***I made one point on a single line. We call a line with one point a one-ple.*** Then the teacher makes two points as in the left photo below and says, ***I made two points on a single line. If we call a line with one point a one-ple, what would we call a line with two points? Yes, a two-ple.*** The teacher makes two points in a different way, as shown in the photo below on the right, and says, ***I made another two-ple in a different way.***

VOCABULARY

The term multi-ple prompts students to see the points on each line as a whole rather than as individual entities. Multi-ples can be created on both vertical lines (VL) and on horizontal lines (HL).

Multi-ples	On a VL	On an HL
Two-ple		
Three-ple		
Four-ple		

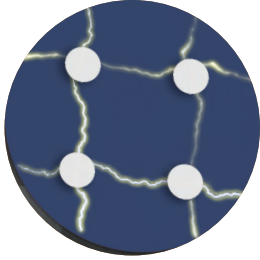


Task

1. Tell students, ***With your partner, your task is to figure out different ways that you can make three points on a single line.***
2. As you circulate, observe and listen to students' conversations. Here are some things to watch for:
   - It is common for students to initially place only three fingers down on either the side, or on the bottom.
   - Students may initially make one point and then press two more fingers.
   - If this strategy persists, tell students, ***You should make three points. Three points should appear at the same time.***

TEACHER TIP

Students must generate three parallel lines first to create a three-ple (single line with three points on it).



Extending Student Learning

Ask, ***How can you make four points on one line? How can you create a line with seven points? I wonder how many points can you create on a single line?***

3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:

    - The goal of this task is for students to develop fluency in performing the **unitising** action with different numbers, using only one multi-ple making finger.

    - If they don't immediately create the four-ple or seven-ple, prompt them to do so.

Summarise

1. Once the pairs have successfully created a three-ple (three points on a line), gather them together for a class discussion. Be sure to have the Zaplify world projected for all to see.

2. Ask for a pair to demonstrate and explain to the group how they created a three-ple. Encourage the pair to verbalise what they did. Prompt students to elaborate upon their explanation with questions such as, ***Did you get this right away? If not, what did you do first? Can you explain how you knew that was incorrect? How did you figure out how to create a three-ple instead of three one-ples?***

3. While a pair is demonstrating how to create a three-ple, ask, ***What will happen if we add another vertical line?*** After sharing a few predictions, have the pair add a vertical line so that all can see what happens. ***What happened to the horizontal line? What do you notice about the number of points?*** While the pair holds their fingers on the screen, ask, ***What will happen if we add another horizontal line?*** After sharing a few predictions, have the pair add a horizontal line and ask, ***What happened to the horizontal line? What do you notice about the number of points? What should we call this? What would a five-ple or a six-ple look like?***

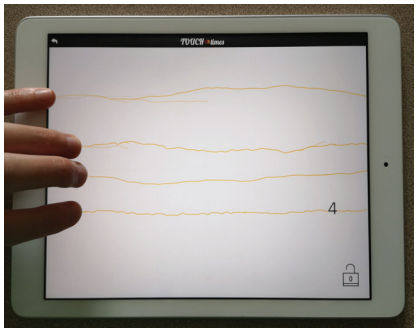
4. Ask, ***Did anyone else find a different way of doing this?*** This provides an opportunity to discuss and demonstrate how 3 x 1 = 3 and 3 = 1 x 3 are both ways of creating a three-ple. ***Does the order of the lines in this case matter? Why or why not?*** Make explicit that the black numerals represent the factors and the orange numeral(s) represents the product.

TEACHER TIP

When there is only one three-ple on the screen, students might equate the product with the multi-ple as a generalisation.

When the teacher adds both a vertical and then a horizontal line, there will be two four-ples on the scren. Describing this as two four-ples, students can identify the difference between a multi-ple and a product.

Assessment

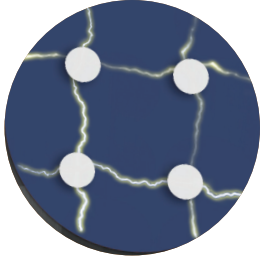


1. Draw how the screen would change if you:
    - place one finger along the bottom
    - place two fingers along the bottom
    - place one finger along the bottom and one finger on the left side.
2. Show how to make 6 intersections in Zaplify in two different ways.

Resource Connections

This task can be connected to arrays and the area model. See the resource connections for the Grasplify task “Many-to-one: Part B” on page 31, which are also applicable for this Zaplify task.

Skip Counting - Part A



Summary

Students focus on how changing the number of lines affects the composition of all the perpendicular lines. This is a more transformational approach to multiplication than repeated addition and focuses on how the intersection points are spreading across every vertical line when a horizontal line is added and vice versa. The addition of one line increases the number of intersection points that correspond to the factors.

Video Link

Curriculum Objectives (for Part A & B)

Big Ideas & Curricular Competencies

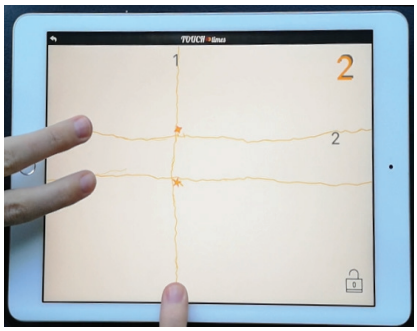
See page 11 for the Big Ideas and the Curricular Competencies for these tasks.

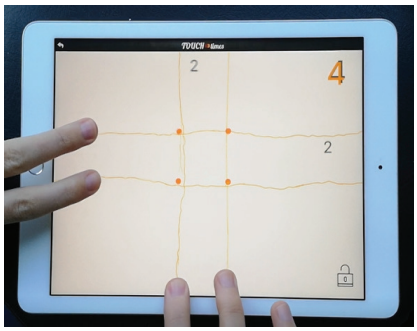
Content

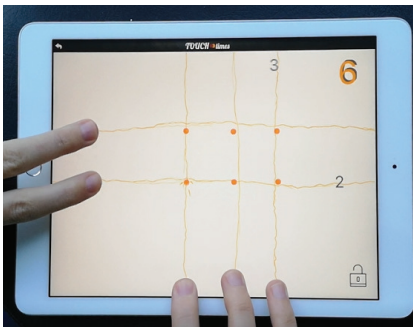
- Connect multiplication to skip-counting
- Connect multiplication to repeated addition

Introduction - Part 1

1. Project Zaplify onto a screen for the class to view. Tell students, ***Last class I made a line with two points by placing two fingers at the side and one finger at the bottom. We called this a two-ple.*** Invite students to make other multi-ples on vertical lines. Show students what it looks like when you change the number of vertical lines. Place one finger at a time on the screen to create vertical lines and say, ***Now, I'm changing the number of vertical lines. Check how the number of intersection points and product changes.*** Students will likely recognise this as skip counting.







Task

1. Say, ***I've shown you how to skip count by two by changing the number of vertical lines. When you get your iPad, your challenge is to find a way to skip count by five by changing the number of horizontal lines. I changed the vertical lines, show me using hand motions, which way is vertical. Now show me using hand gestures which way is horizontal. I skip counted by two by changing the vertical lines. Your challenge is to skip count by five by changing the horizontal lines.***
2. Allow time for students (alone or in partners) to work on the task.
3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:
    - Skip counting by 5 by changing the number of horizontal lines will require students to place 5 fingers on the bottom of the screen and 1 finger on the side of the screen. This creates 5 points on the horizontal line, and by placing one more horizontal-finger at a time, Zaplify will skip count by 5.
    - If students persist with skip counting by changing the vertical lines, ask them to make 5 in a different way. This may prompt them to start by making five vertical lines and one horizontal line.
    - You may also want to ask students to record the successive products that they discover in their notebooks. They could also draw pictures to show what each product looks like in Zaplify.

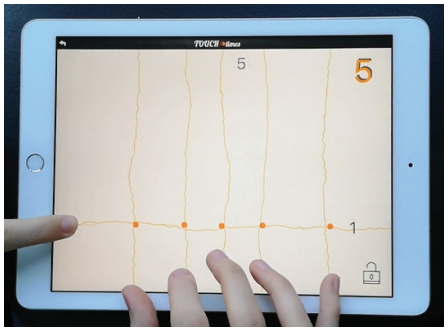
WHAT TO WATCH FOR

Ensure that students are skip counting by adding horizontal lines. We are intentionally holding the number of vertical lines constant and seeing what happens by adding more horizontal lines.

Extending Student Learning

As you walk around and see that some students have succeeded in skip counting by fives by changing the number of horizontal lines, you can invite them to try skip counting by 3, 7 or 10. You might also invite the students to skip count by a larger number that they are not as familiar with, and to record the products in their notebooks.

Introduction - Part 2



Make a five-ple on a horizontal line in Zaplify so that all students can see it projected on the screen (see picture on the right). ***This has been made using five vertical lines and one horizontal line and it is called a five-ple. How can I skip count up to 25 by five-ples?*** Following student explanations, demonstrate how to do this by creating additional horizontal lines. With 25 still projected for all students to see, ask, ***Does anyone have an idea about how we could skip count backwards by 5?*** When a student suggests to remove one of the fingers creating a horizontal line, ask, ***Who has a prediction about how the product will change when I lift my finger?***

Task

1. Say, ***Starting with a product of twenty-four, how can you skip count backwards by only changing the horizontal lines so that you end up with a product of twelve?***
2. Allow time for students (alone or in partners) to work on the task.
3. As you circulate, observe and listen to students' conversations. This could be done in several ways. Student can skip count backwards by:

Skip Counting by:	Start With:		Skip Counting Pattern
	Horizontal Lines	Vertical Lines	
12	2	12	24-12
6	4	6	24-18-12
4	6	4	24-20-16-12
3	8	3	24-21-18-15-12
2	12	2	24-22-20-18-16-14-12

Extending Student Learning

Say, ***Make a product of fifteen in Zaplify and skip count backwards by three by only changing the number of horizontal lines.*** Students will first need to decide how to make the product - 3 vertical lines, 5 horizontal lines, not the other way around.

Summarise

1. Once the pairs have successfully completed the task, gather them together for a class discussion. Be sure to have Zaplify projected for all to see.

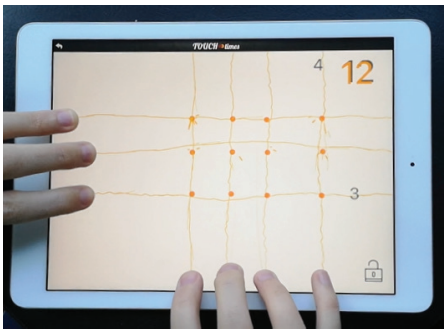
2. Ask for a pair to demonstrate and explain how they made 24 and what they did to skip count backwards to 12. Encourage the pair to verbalise what they did. Prompt students to elaborate upon their explanation with questions such as, ***How did you figure out how to make the product twenty-four? Which factors did you use? Did you get this right away? If not, what did you do first? How did you know that was incorrect? How did you decide which lines to use for counting backwards? What is the name of that m-ple that you were lifting?***

3. Ask, ***Did anyone else find a different way of doing this?*** This provides an opportunity to discuss counting backwards with other multi-ples. ***I want to skip count backwards by threes by changing the number of horizontal lines. How should I make twenty-four in this case? Which factors should I use? Where should I put my fingers?***

Assessment

Project the image below.

Ask students to complete the following:



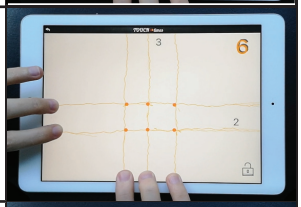
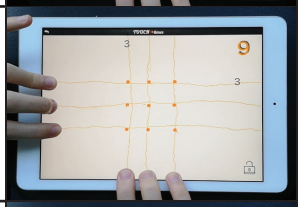
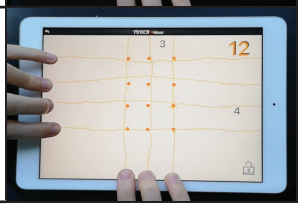
- ***Sanna used Zaplify to skip count to twelve. Describe each of the steps that she did if she was skip counting by changing the horizontal lines.*** If students need a hint, ***Sanna started with just four vertical lines.***

Resource Connections

A common problem that requires skip counting is: ***There are 3 wheels on a tricycle. How many total wheels are there if there are 5 tricycles?***

Linking cubes can be used to model this situation. Students create groups of three linking cubes to model a tricycle and make five of these groups of three to find the answer. In Zaplify, they can model this problem by pressing three fingers on the edge and 5 sequential touches on the bottom of the screen, so the answer is 15. Zaplify allows the opportunity to change tricycle to bicycle (change 3 to 2), or tricycle to car (change 3 to 4) by simply changing the horizontal lines. Zaplify offers a more dynamic and fluent process to draw attention to the **spread** of intersection points across the vertical lines (or vice versa).

This task can also be connected to tables. The table highlights the functional view of multiplication - in this case, the function is to triple. The first example shows that the relation between the number of horizontal lines (1), which has three points on it (three-ple), the product (3) and the number of vertical lines (3). The number of horizontal lines has increased by 1 in each row. The functional relation is being emphasised because it shows that if 1 three-ple makes 3, then 4 three-ples makes 12. This table can be extended by changing the number of horizontal lines.

Number of Three-ples	Product	*TouchTimes* Image
1	3	
2	6	
3	9	
4	12	

Skip Counting - Part B

Video Link

Summary

Students continue to investigate the idea of **spreading**, which is more multiplicative in nature than repeated addition.

Introduction

1. Project Zaplify onto a screen for the class to view. Tell students, ***Last class you skip counted backwards. Who can remember one way to skip count backwards from twenty-four to twelve?*** As students explain how to skip count backwards by 2, 3, 4, 6 and 12, demonstrate the process to skip count by each multi-ple. You may need to remind them of the terms two-ples, three-ples, four-ples, six-ples and twelve-ples.

2. Ask, ***Is it possible to count backwards from twenty-four by using five-ples?*** Try students suggestions and say, ***It is not possible to count backwards starting from twenty-four by using all multi-ples.***

Task

MAKING 198 WITH:	
Two-ples	
Three-ples	
Four-ples	
Five-ples	
Nine-ples	

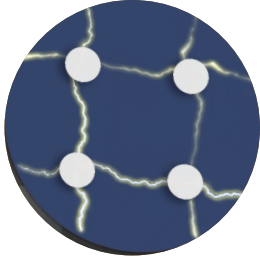
1. Say, ***Today I want you to try to make the product 198 by counting up with the multi-ples on this chart. You can try to skip count using two-ples, three-ples, four-ples, five-ples and nine-ples. There's a trick though, you can only reach 198 with only three of these multi-ples and you need to figure out which three.***

2. Allow time for students (alone or in partners) to work on the task.

3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:

   - Students can create multi-ples either on horizontal lines or on vertical lines.

   - Students might tap very quickly to increase the product. Prompt them to slow down as the product becomes larger.



Extending Student Learning

- Additional multi-ples (six-ples, seven-ples, eight-ples, ten-ples) can be given to early finishers.
- Ask, ***Are there any that you know immediately will not work? How do you know?***

Summarise

1. Once the pairs have completed the table, gather everyone together for a class discussion. Be sure to have Zaplify projected for all to see.
2. Ask, ***Could you reach 198 with two-ples?*** Ask a pair to demonstrate and explain how they made 198 using two-ples.
3. Ask, ***Which other multi-ples did you use to make the product 198?*** Have pairs come to the front to demonstrate and explain how they made 198 using other multi-ples.
4. Repeat this with five-ple, ***Could you reach at 198 using five-ples? Why not?*** Ask for a pair to demonstrate and explain to the class why they could not make 198. Encourage the pair to verbalise what happened when s/he tapped the last time. Prompt students to elaborate upon their explanation with questions such as, ***What was the product before your final tap? How many more points do you need? Can you create that many points with one touch? How many points was your finger creating with one touch?***
5. Explain, ***Did anyone notice a pattern while skip counting with five-ples?*** Wait for some responses. ***When skip counting by five-ples, the ones digit will always be either zero or five. How does this help me know immediately if I can make 198 with five-ples? What about ten-ples? What about two-ples?***

Resource Connections

This task can be connected to skip counting on a number line. See the Resource Connection for the Grasplify task “Skip Counting by Pips” described on pages 34-35.

TOUCH times

Video Link

Doubling

Summary

Students create pictorial representations of multiplication using *TouchTimes* to investigate the mathematical strategy of doubling. In this task students explore the idea of simultaneous multi-plying, simultaneously creating a double of an original quantity.

Curriculum Objectives

Big Ideas & Curricular Competencies

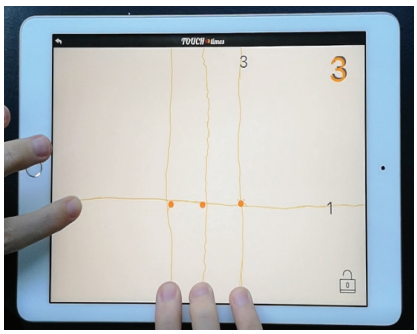
See page 11 for the Big Ideas and the Curricular Competencies for these tasks.

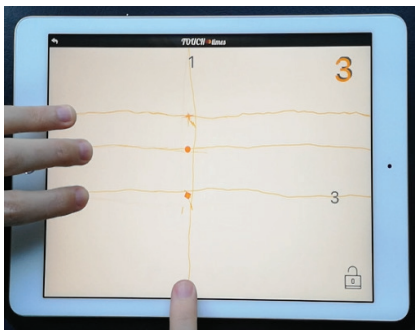
Content

- Provide opportunities for concrete and pictorial representations of multiplication
- Using mental math strategies such as doubling
- Fluency with math strategies (e.g. identifying related doubles)
- Understanding concepts of multiplication (e.g. arrays)

Introduction

Explain, ***We have been learning a lot about multi-ples in the last few tasks. How can I make a three-ple?*** As students explain how to do this, create a three-ple using *TouchTimes* so that all students can see it projected on the screen. ***What product do I have with a single three-ple? What are the factors? Is it possible to get a product of three another way?*** Emphasise that in order to make a product of 3, you need to create a three-ple by making one lightning bolt, either vertically or horizontally, and three lightning bolts in the other orientation (either vertically or horizontally).

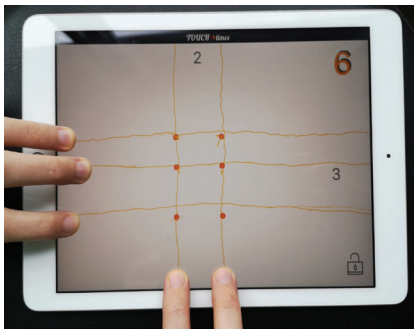


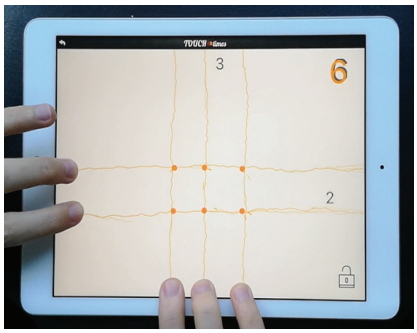


Task

1. Explain, ***Your challenge today is to double a product of three so that it is six. First, make a product of three. How can you double the product so that it is six by changing the number of horizontal lines OR the number of vertical lines? Once you've done that, double the product again to make twelve. Then double it again to make twenty-four.*** Write the 3, 6, 12, 24 sequence on the board for students to refer to while working.

2. Allow time for students (alone or in partners) to work on the task.





TEACHER TIP

An important aspect of multiplying is the simultaneous action as compared to sequential adding. You may want to encourage students to double the product in one action instead of multiple actions.

Extending Student Learning

- Early finishers can be challenged with, ***Can you start with a product of five and double it?***
- Another extension is to invite students to triple the product. They will quickly run out of fingers, so this might also be a task where they can imagine how to triple the product and document their thinking on paper.
- Early finishers can also be challenged to explore some of the I Wonder questions from the anchor chart that they created during the Zaplify exploration session on page 42.

Summarise

1. Once the pairs have successfully doubled three to create products of six, twelve and twenty-four, gather everyone together for a class discussion. Be sure to have Zaplify projected for all to see.

2. Choose a pair to create 3 on the iPad that is being projected for all to see. Have the pair demonstrate and explain how they doubled the product to make 6. Encourage them to verbalise that the placement of their fingers. Ask, ***Does anyone double the product of three in a different way? Come and explain us.*** Try to choose the students who made the 3 with three vertical lines and one horizontal line. ***So in this example, when we increased the number of lines to double the product, what was happening that caused the product to double? How do the factors change when you double the product?***

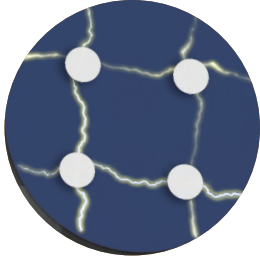
Assessment

Show or explain how you would solve this problem using Zaplify:

There are six lanes in a swimming pool. In Mathew's swimming lesson, every lane is shared by two students. When another class joins Mathew's lesson, the number of students that share a lane is doubled. How many students are swimming in the pool now?

Resource Connections

This task can be connected to arrays. See the Resource Connection for the Grasplify task "Doubling" described on page 20.



Halving

Summary

Students create pictorial representations of multiplication using *TouchTimes* to investigate the mathematical strategy of halving. In this task students explore the idea of multi-plying, simultaneously creating half of an original quantity.

Video Link

Curriculum Objectives

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for these tasks.

Content

- Provide opportunities for concrete and pictorial representations of multiplication
- Using mental math strategies such as halving
- Fluency with math strategies (e.g. identifying related doubles)
- Understanding concepts of multiplication (e.g. arrays)

Introduction

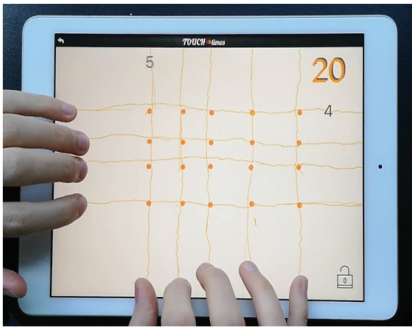
Tell students, ***Last class we started with a product of three and doubled it to make a product of six, which we then doubled to make twelve, which we doubled to make a product of twenty-four.*** Double the three-ple and ask, ***How many three-ples does six include?*** Repeat this with twelve and twenty-four.

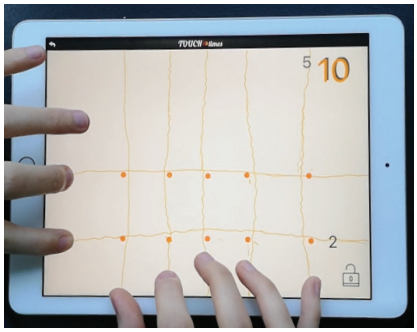
Task

1. Using Zaplify, make the product twenty with factors 4 and 5so that all students can see it on the screen. Say, ***Right now, the product is twenty, how can you halve the product so that it is ten? Once you've done that, halve the product again to make five.*** Write the 20, 10, 5 sequence on the board for students to refer to while working.
2. Allow time for students (alone or in partners) to work on the task.

3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:

    - Students will have to shift from adding more fingers to removing half of their horizontal fingers each time.





    - Some students might lift their fingers one by one. If that happens, prompt them to lift their fingers all at once. Encourage them to decide how many fingers they will lift before lifting any fingers. For example, if the student creates twenty, as in the photo above, you can point to the horizontal lines and ask, ***How many fingers do you need to lift to halve this factor? Let's see how this changes the product.***

Extending Student Learning

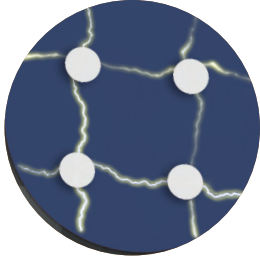
- As pairs successfully complete the given task, challenge them to find other ways to halve the product. This will prompt them to work on an impossible task that is halving the product of 20 by halving the factor of five.
- Early finishers can be challenged to try halving the product of 40 in different ways.
- Students can also be asked to repeatedly halve the product starting from 50, which would quickly produce a challenge (halving 25).
- Be sure to try a challenging question like 3 × 7 and ask about halving.

Summarise

1. Once the pairs have successfully completed the task, gather everyone together for a class discussion. Be sure to have Zaplify projected for all to see.
2. Create 20 on Zaplify by making 4 horizontal lightning bolts. You may want to pretend not to know the answer to encourage students to explain what to do. Say, ***What do I have to do next in order to halve the product so that it becomes ten?*** Choose students to explain

TEACHER TIP

Challenging questions direct student attention to reunitising (for 21 we would need 10.5 units of 2).



what you need to do. If they say lift two fingers, you may want to question, ***Which side?*** As they explain, ***Hey, that worked! How did you know that? Now what do I do?*** Continue to have students explain exactly what to do and question how they know.

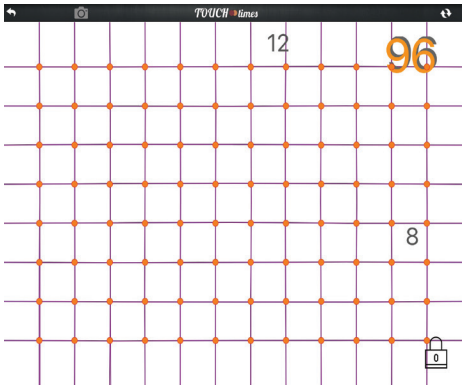
3. Do the same by creating 20 by making 5 horizontal lightning bolts. Ask students, ***Why can't I halve twenty by changing the number of horizontal lines? Is there another way to halve twenty?***

4. Ask, ***How is doubling different from halving?***

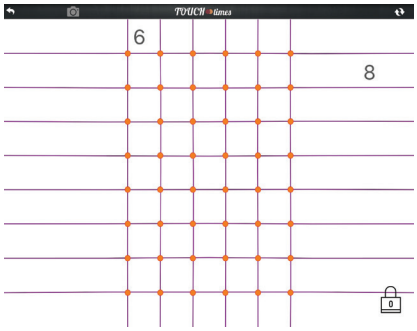
Assessment

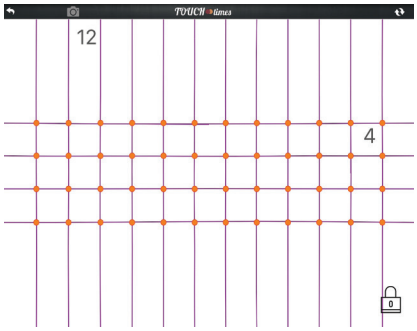
1. If you know that 48 x 12 = 576, how would you use that information to find the answer to:

   - 48 x 6
   - 24 x 12



2. How would you find the product in the pictures below by using picture above?





Resource Connections

This task can be connected to arrays. See the Resource Connection for the Grasplify task "Halving" described on page 24.

Making the Larger Product – Part A

Video Link

Summary

Students investigate the covarying relationship between each factor and the product by changing the factors and observing how each factor changes the product differently. By comparing how the factors interact to increase or decrease the product, students try to determine how to make the larger product.

Curriculum Objectives (for Part A & B)

Big Ideas & Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for these tasks.

Content

- Using flexible computation strategies (e.g. decomposing)
- Understanding concepts of multiplication (e.g. arrays)

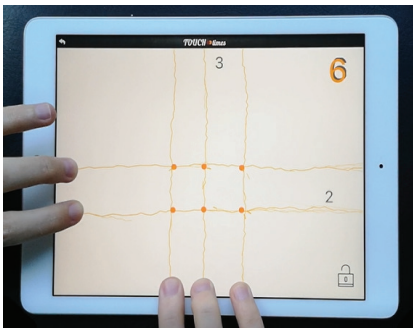
Introduction

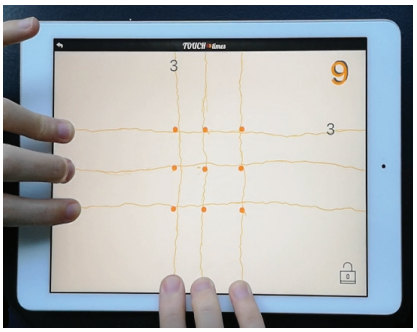
Tell students, ***Last class we started with a product of twenty and kept halving it until we reached five. How did we place our fingers to do that?*** Model student suggestions using Zaplify so that all can see. ***By halving, we decreased the product. Today we will make the product larger with one touch. The trick in this task is to figure out how to make the product larger with only ONE touch.***

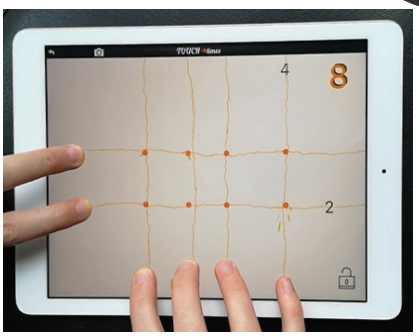
Task

1. Say, ***Make a product of six by making the factors two and three. If your partner could place one more finger on the screen to increase one of the factors, which factor should they increase to make the larger product? Before you try it though, I want you to predict which factor should be increased and what will the new product be. Then try it out to see if your prediction was correct. Does it matter if your prediction is wrong? No, but if your prediction was incorrect, try to figure out why. If your prediction was right, how did it work?***

2. Allow time for students (alone or in partners) to work on the task.







3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:

   - Remind pairs to make their predictions first and then try it out on Zaplify.
   - Some students may lift their fingers as soon as they place them on the screen. Prompt them to hold their fingers on the screen until the end of the task.
   - Once students have created 2 x 3 = 6, if they add multiple fingers, prompt them to start over, reminding them that they can only use only one finger in this task.

Extending Student Learning

- Early finishers can be challenged with a more open task, ***Place as many fingers as you want on one side of the screen. Your partner will place as many fingers as they want on the other side of the screen. Which factor should you increase by one to make the larger product?*** Students can continue to play this game by taking turns.
- You can extend this task by asking students, ***Place two fingers along the side and three along the bottom to produce six. Your partner can put two more fingers on the screen. Where should they be placed to make the product larger? Where should they be placed to make the smaller product?***

Summarise

1. Once the pairs have successfully maximised the product, gather them together for a class discussion. Be sure to have the Zaplify world projected for all to see.

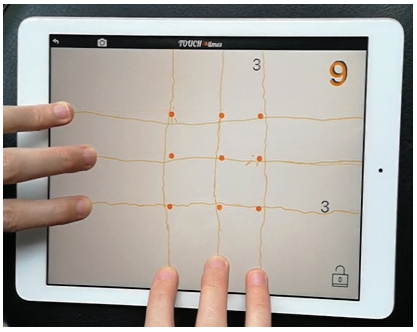
2. Have a pair create 2 x 3 = 6 in Zaplify and then demonstrate how they made the larger product with one finger. Encourage them to verbalise where they placed the additional finger. Ask students, ***What happened when [insert student's name] made a horizontal line?*** In order to draw attention to the multi-plying aspects of multiplication, ensure that children explain what happened on the vertical lines when another horizontal line was created. ***How many new points appeared on each vertical line? How many new points in total appeared? Is there another horizontal line with the same number of points? Do you notice anything about the three-ples?*** Draw attention to the number of points on each horizontal line and the relationship between the points and the number of vertical lines.

3. Explain, ***So in this activity when we increased this factor (point to the horizontal lines) to make the larger product, what was happening that caused the product to increase more?***

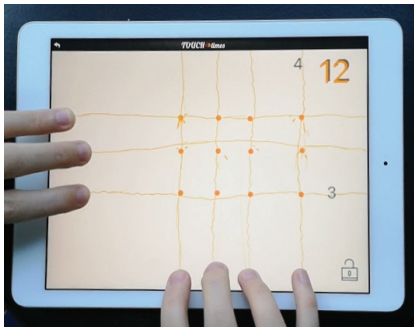
4. While projecting 3 x 3 on the screen, ask, ***Where should you place one finger to make the larger product now?*** Encourage students to verbalise where they would place the additional finger.

TEACHER TIP

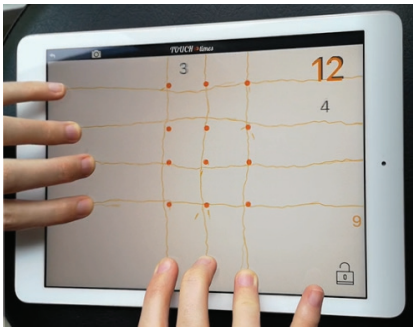
We want students to notice the relationship with the multi-ples and how the product is influenced by the addition of a vertical line or a horizontal line.



A Square Number



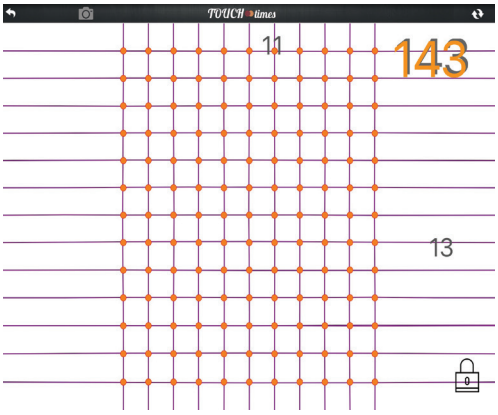
1 More Vertical Lightning Bolt



1 More Horizontal Lightning Bolt

5. Explain, ***In this activity it didn't matter whether we increased the horizontal lines by one or the vertical lines by one to maximise the product. These products are called square numbers. Can someone tell me another square number?***

Assessment



1. You know that 13 x 11= 143.

   - How will the product change if you increase the first factor from 13 to 14?
   - How will the product change if you increase the second factor from 11 to 12?

Explain your answer by drawing on the given picture.

Making the Larger Product – Part B

Video Link

Summary

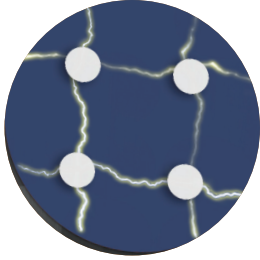
Students continue to create square numbers. This task is an arithmetic version of what students will later see in algebra, when they work with the product of binomials.

Introduction

Tell students, ***Last class we created square numbers that we could maximise by adding either a horizontal line or a vertical line. It didn't matter which one.*** Demonstrate a few square numbers such as 4, 9, and 16 and ask, ***How much does the product increase when I increase one of the factors? Why? Today we will jump from one square number to another one.***

Task

1. Say, ***Place two fingers along the left side and two fingers along the bottom to produce four and then increase each factor by one. Draw a sketch of what you see. Then use Zaplify again to make 3 x 3 and increase each factor by one. Draw another sketch of what you see. How many new points do you predict will appear if you increase each factor by one in 4 x 4? Draw a picture showing your prediction.***
2. Allow time for students (alone or in partners) to work on the task.
3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:
    - Prompt students to work as a pair by taking turns. While one student holds their fingers and increases the product, the other student draws a sketch. Prompt students to use pencil crayons to depict the intersection points.
    - Some students might add lines to an existing sketch. If that happens, prompt them to make a separate sketch for each situation and compare all of them at the end. They can make their drawing on the iPad, using the built-in drawing app—they might even want to start with a screen image of, say, 3 x 3, and simply add on the required elements.



Extending Student Learning

- Early finishers can be asked to add more than one finger to the factors and to predict what would happen by making a quick sketch.

Summarise

1. Once the pairs have successfully finished their sketches, gather them together for a class discussion.

2. Have three pairs share their sketches. Assign different square numbers to the pairs to share. Ask each pair, ***When you added a finger how many more intersection points did this create?*** Repeat students answers by saying, ***The first group jumped from four to nine. The first finger created two new points, the second finger created three more points.*** Do this for each group and ask, ***I see a relationship between the number of points that are created by each finger. Do you see a relationship, too?*** Invite students to share their ideas.

3. Explain, ***The second finger creates one more point than the first finger does. What do you think caused this additional point?*** Draw students' attention to the intersections between the lines.

TEACHER TIP

We want students to notice that there's an extra orange intersection point that must be counted, when we add the second finger, which is the result of the covarying of the factors.

Maximising the Product

Video Link

Summary

Students continue to investigate the covarying relationship between each factor and the product by changing the factors and observing how each factor changes the product differently. In this task, students will explore how to maximise the product (make the largest product possible) when there is a restriction on the number of fingers.

Curriculum Objectives

Big Ideas and Curricular Competencies

See page 11 for the Big Ideas and the Curricular Competencies for these tasks.

Content

- Understanding concepts of multiplication (e.g. arrays)
- Looking for patterns in numbers to further develop understanding of multiplication computation.

Introduction

1. Tell students, ***Last class we jumped from sixteen to twenty-five and calculated how many points we needed to jump. Do you remember how many new points appeared when you pressed one more finger on each side?*** Give some time for students to share their answers and ask, ***How many fingers did you use to produce twenty-five?***

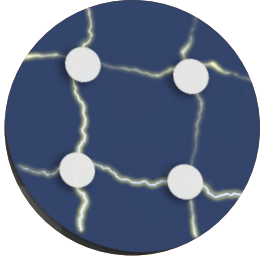
Task

1. Say, ***Using exactly ten fingers, is it possible to make a product larger than twenty-five?***

2. Allow time for students (alone or in partners) to work on the task.

3. As you circulate, observe and listen to students' conversations. Here are some things to watch for:

    - Some students might have difficulty starting the task. If this happens, prompt them to make a product with ten fingers and then change the factors to try to get a bigger product. Ask them to try



different combinations by changing the locations of their fingers.

- Some students might use less than ten fingers. If that happens, prompt them to press exactly ten fingers down.
- Some students might aim to produce 10 by using less than ten fingers. If that happens, explain that the total number of fingers should be 10, not the product.

Extending Student Learning

- Early finishers can be asked to make the largest product using only 11 or 12 fingers. They might also try to predict the largest products for larger numbers, without using Zaplify.

Summarise

1. Once the pairs have successfully finished making several products, gather them together for a class discussion. Have a pair to share their findings. Encourage them to verbalise which factors they use to make different products. Ask the pairs, ***What is the smallest product? With which factors did you produce it?*** W***hat is the largest product? Which factors did you use?***
2. Explain, ***I see a relationship here. Do you see any connection between the factors and the size of the product? How does the difference between the factors change as the product increases?***

Resource Connections

This task can be connected to a specific type of geometrical problem known as maximising the area. An example problem might be “Judy has 22 meters of fencing and she wants to enclose a community garden for her school. What is the largest space that she can enclose with her fencing?” When students are given this type of problems, they can come up with a rule: “The rectangle with the largest area for a given perimeter will be a square”. You can also have students complete this problem using grid paper.

TEACHER TIP

We want students to notice that the product is the biggest when the difference between the number of fingers on both sides is the smallest.

Assessment Ideas

Drawing

Students can be asked to show their understanding of many tasks through drawing. Depending on the task, a single drawing may be sufficient, or students can be provided with a comic strip panel in order to encourage a sequence of drawings that demonstrate their understanding. An example prompt that could be used for the Doubling task in either Grasplify or Zaplify could look like this:

Make a comic strip drawing that shows what the Grasplify (or Zaplify) screen would look like if you start with 1 x 4 = 4. Show how it would change if you double the product to 8 by only changing the number of pips (or horizontal lines). Be sure to use your pencil crayons to show the colours. If you would like to continue doubling the product using pips (or horizontal lines), you can add more frames to your comic strip.

Another example of using drawing for assessment can be found on page 35 for the Pips vs Pods task.

Written Response

Students can demonstrate their understanding by providing written instructions to someone who has never used *TouchTimes* before, about how to complete one of the tasks. This assessment idea can easily be adjusted for either Grasplify or Zaplify. An example prompt that could be used for the Halving task could look like this:

Provide step-by-step instructions for someone who has never used Grasplify (or Zaplify) before.

1) Explain how to create 12 = 3 x 4. Don't forget to use the words pips and pods (or vertical and horizontal) in your explanation. You can use RS for the right screen and LS for the left screen.

2) Explain how to halve the product by only changing the pips (or the vertical lines).

Audio and Screen Recording Apps

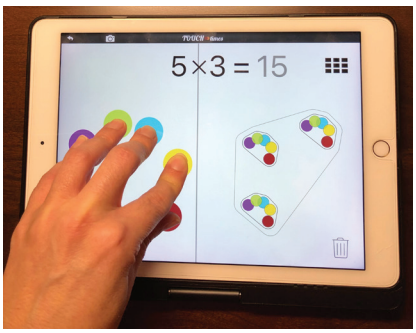
Using an audio and screen recording app, such as Show Me, students demonstrate and explain their understanding. An example prompt that could be used for Many-to-One (Part A) and Skip Counting by Pips could look like this:

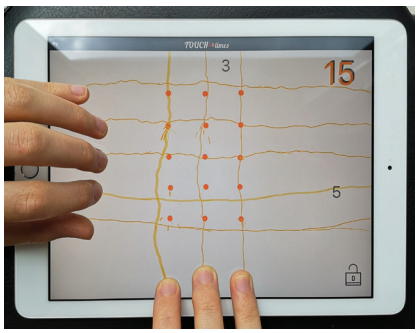
Using the Show Me app, demonstrate and explain to your parents how to create a 4-pod and then skip count by pods. Once you've done that, demonstrate and explain how you can also skip count by fours in Grasplify using only the pips.

More Assessment Ideas

Screen Shots

Screen shots of *TouchTimes* can be used to prompt student responses that can then be used for assessment. Student responses can include drawings, written or verbal explanations. Students can be asked to explain what is happening in the screen shot or can also be promptd with “what would happen if...” scenarios. Example prompts could look like this:





Left photo: Katie creates 5 x 3 = 15 using Grasplify, and it looks like this. Draw what would happen if Katie lifted a finger off the iPad or draw what would happen if Katie placed two more pip-making fingers on the left screen.

Right Photo: Katie creates 5 x 3 = 15 using Zaplify, and it looks like this. Draw what would happen if Katie lifted a finger off of the side or draw what would happen if Katie placed two more fingers on the bottom.

Word Problems

Students can be given word problems and asked to show how these using either Grasplify or Zaplify.

A couple examples of word problems that could be used are:

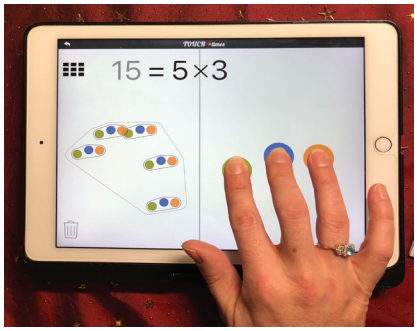
Some kids were playing a team game. Each team has 3 kids and there are 4 teams. Show this using Grasplify.

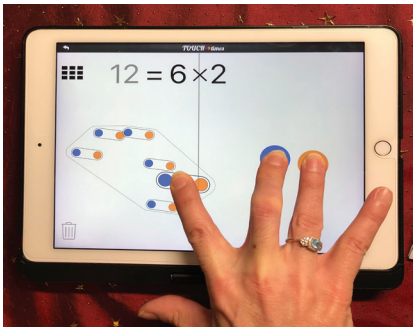
I was in a classroom where desks were organized into rows. Each row had 6 desks. One classroom has enough space for 5 rows of desks. How many desks would be in 2 classrooms? Use Zaplify to show your answer.

Additional Task Ideas

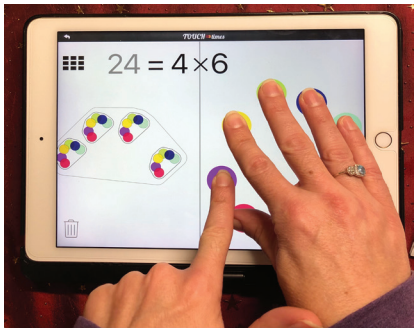
Grasplify

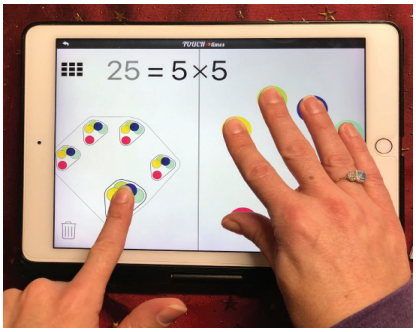
Task: ***Make 3 x 5 = 15. What do you think will happen if you move one pip-making finger to the other side of the screen? Will the product be bigger or smaller?***





Task: ***Make 6 x 4 = 24. What do you think will happen if you move one pip-making finger to the other side of the screen? Will the product be bigger or smaller?***





Class Discussion: Have students predict what will happen with other examples, 5 x 5, 2 x 6, 6 x 2. Lead the group to generate a more general statement, such as "*the product will increase if the number of pips is smaller than the number of pods*" and perhaps even try to explain why that happens, and what happens when the number of pips and the number of pods are the same.

Task: ***Make 5 x 5 = 25. Re-create that product and press the array button. Make a sketch of what you see displayed on the iPad. Now make two other products that use exactly 10 fingers and draw what you see displayed. Compare the drawings you've made. How are they the same? How are they different?***

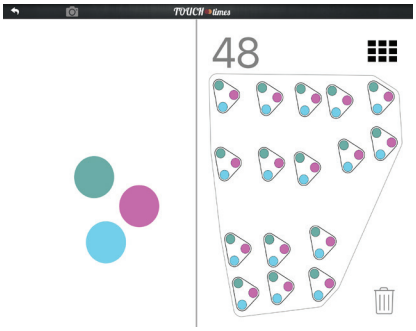
Ensure that student drawings are detailed enough to show the dimensions and size of each array. The goals here are to introduce the array button and to draw attention to the shape of the array, and to the fact that different products produce different arrays/shapes.

Task: ***How can you make the largest product with only 11 or 12 fingers?***

Class Discussion: Ask students what they've discovered about maximising the product. You are looking for descriptions such as 'the product is biggest when the difference between the number of pips and the number of pods is smallest'.

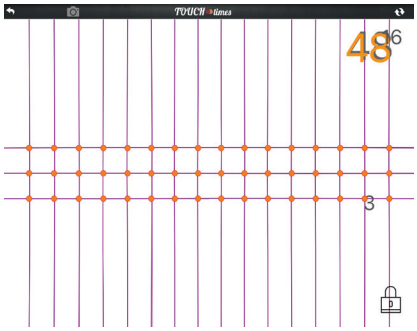
For more exploration, ask students whether it is possible to use less than 10 fingers to make a product that is bigger than the ones they made with exactly 10 fingers. Children could be invited up to the front to try these out. For example, 4 x 4 = 16 only uses eight fingers, but it is greater than 1 x 9 = 9, which uses 10 fingers.

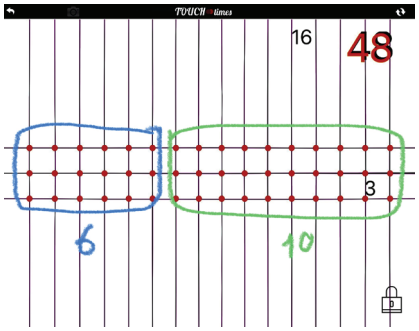
Task: (With numeric factor values setting turned off) ***In the image below, Kumari made 3 x 16, arranging the pods so that she could see 10 of them towards the top of the screen and 6 towards the bottom. That helped her figure out that 3 x 16 was actually just 3 x 10 added to 3 x 6. Use the same strategy to find the following products: 5 x 14, 4 x 23, 6 x 15, 3 x 29 and take a screen shot of how you decomposed the pod.***



Zaplify

Task: ***In the image below, Kumari made 3 x 16 and decomposed the product into two parts by drawing blue and green boxes. That helped her figure out that 3 x 16 was actually just 3 x 10 added to 3 x 6. Use the same strategy to find the following products: 5 x 14, 4 x 23, 6 x 15, 3 x 29 and take a screen shot and decompose them by drawing boxes.***

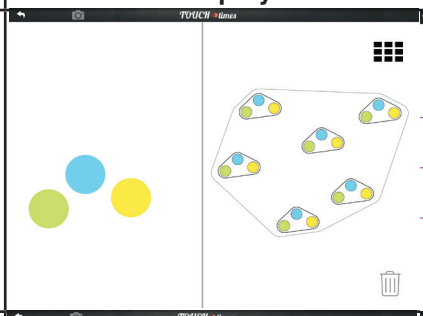
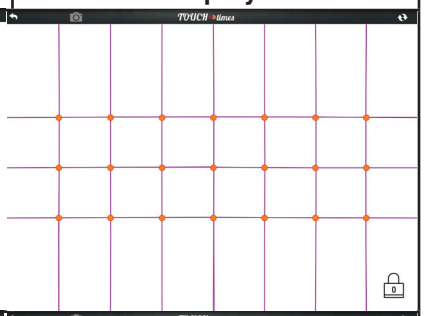
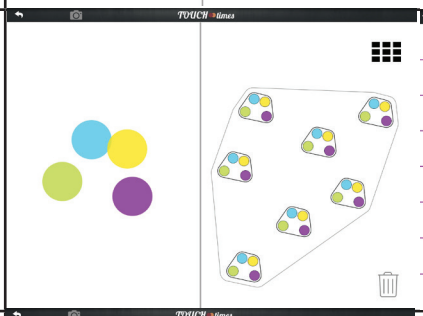
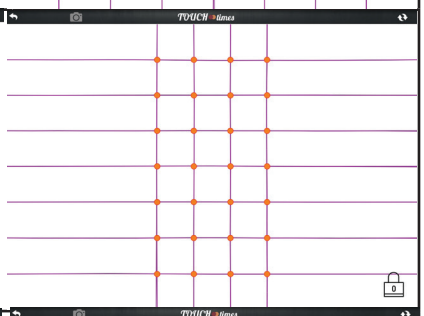
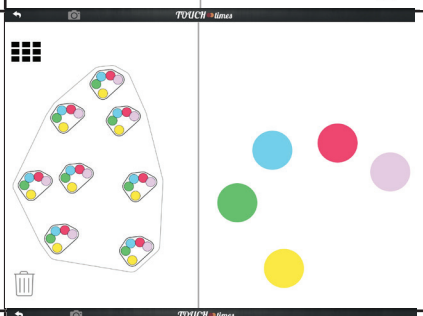
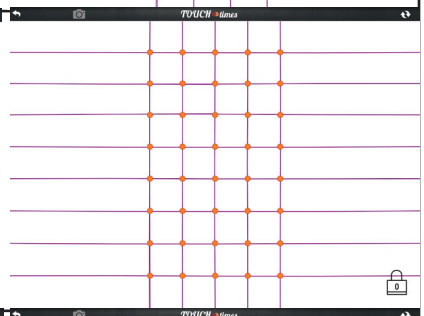
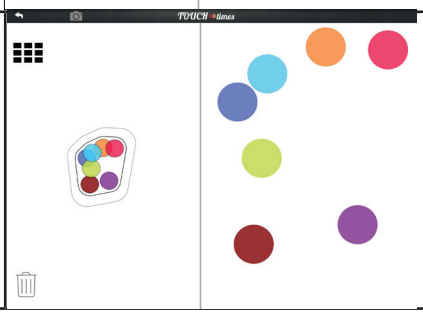
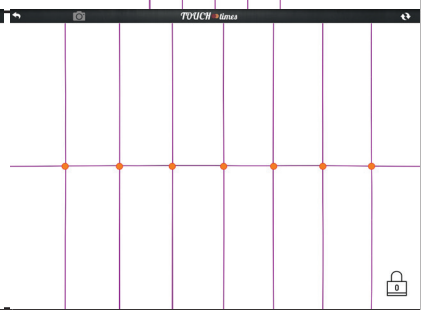


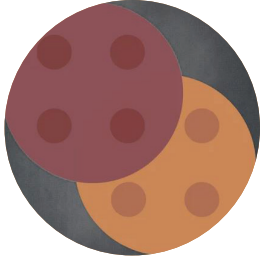


Grasplify or Zaplify

The tasks included in this section can be used effectively with either Grasplify or Zaplify.

Task: (off-line) ***The equation is missing. Write down what equation should have appeared in*** **TouchTimes.**

Equation	Grasplify	Zaplify
Grasplify: 3 x 7 =21 Zaplify: 3 x 7 = 21 7 x 3 = 21 21 = 3 x 7 21 = 7 x 3		
Grasplify: 4 x 7 =28 Zaplify: 4 x 7 = 28 7 x 4 = 28 28 = 4 x 7 28 = 7 x 4		
Grasplify: 40 = 8 x 5 Zaplify: 5 x 8 = 40 8 x 5 = 40 40 = 8 x 5 40 = 5 x 8		
Grasplify: 7 = 1 x 7 Zaplify: 1 x 7 = 7 7 x 1 = 7 7 = 1 x 7 7 = 7 x 1		



Task: *Show how you would solve these problems using* TouchTimes:

(1) A store sells crayons in packages of 6. You decide to buy 3 packages. How many crayons will you have?

(2) You watched 4 seasons of a show and each season had 7 episodes. How many episodes did you watch in all?

(3) A school has 20 clocks. They came in packages of 4. How many packages were bought?

(4) There are 36 kids that have been organised into 9 groups. How many kids are in each group?

(5) There are 24 students in a class. Their teacher wants to arrange their desks into equal groups. What arrangements can be made so that each group has the same number of desks in it?

Task: Students work in partners. Partner 1 creates a multiplication expression (in Grasplify using pips and pods or lightning bolts in Zaplify) that Partner 2 is unable to see. Partner 1 then describes what they have created to Partner 2, who must recreate it on their own iPad.

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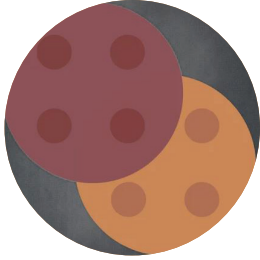

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